Curvature perturbation in multi-field inflation with nonminimal coupling

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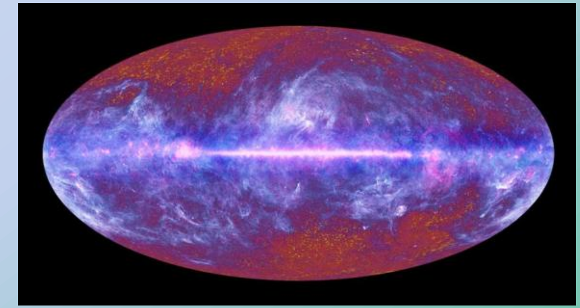
Inflation

Inflation has become the most important element in modern cosmology.

- ➢ explaining flatness, homogeneity and isotropy of the Universe



- ➢ solving the horizon and monopole problems

Guth (81), Sato (81)

- ➢generating the seed of the cosmic structure

Mukhanov (81)

Various modelsclassified from various points of view

- ☐ Prototype modes: New, Chaotic, Hybrid infltion
- ☐ Kinetic terms : Canonical (slow-roll) inflation, k-inflation, DBI inflation…
- ☐ Gravity sector : Einstein gravity, Modified gravities…
- ☐ Inflaton sector: SM Higgs, Superpartners of SM, String moduli…
- ☐ Number of fields: Single- or Multi-field model

We will present formulation of the cosmological perturbations
in multi-field inflation with nonminimal couplng,
recovering some of above models in the corresponding limits.

Many models can be brought into the Einstein gravity through the proper conformal transformation.

- f (R) gravity

$$S = \int d^4x \sqrt{-g}\, f(R)$$

- Non-minimal coupling

$$S = \int d^4x \sqrt{-g} \left(f(\phi) R + \cdots \right)$$

- String-inspired models

$$S = \int d^D x \sqrt{-g_D}\, \frac{e^{-\phi}}{2\kappa_D^2} \left(R_D + \cdots \right)$$

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \quad \Longrightarrow \quad \int d^4x \sqrt{-\tilde{g}} \left(\frac{\tilde{R}}{2\kappa^2} + \cdots \right)$$

Calculations are often more tractable in the Einstein frame than in the Jordan frame.

□ Equivalence of perturbations in the single-field inflation model

Following the notation in Tsujikawa & Gumjudpai (04)

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2} f(\phi) R - \frac{1}{2} \omega(\phi) (\nabla \phi)^2 - V(\phi) \right]$$

Cosmological perturbations

$$ds^2 = -(1+2A)dt^2 + 2a(t)B_{,i}dx^i dt + a(t)^2 \left[(1+2R)\delta_{ij} + 2E_{,ij} \right] dx^i dx^j$$

giving observational predictions, but …

Moving to the Einstein frame

$$\Omega^2 = f$$

$$d\tilde{\phi} = d\phi \sqrt{\frac{3}{2} \left(\frac{f'}{f} \right)^2 + \frac{\omega}{f}}$$

$$S = \int d^4\tilde{x} \sqrt{-\tilde{g}} \left[\frac{1}{2} \tilde{R} - \frac{1}{2} (\tilde{\nabla} \tilde{\phi})^2 - \frac{V}{f^2} \right]$$

In the Einstein frame

$$d\tilde{s}^2 = -\left(1+2\tilde{A}\right)d\tilde{t}^2 + 2\tilde{a}\left(\tilde{t}\right)\tilde{B}_{,i}dx^i d\tilde{t} + \tilde{a}\left(\tilde{t}\right)^2\left[\left(1+2\tilde{R}\right)\delta_{ij} + 2\tilde{E}_{,ij}\right]dx^i dx^j$$

The background and perturbation variables are related

$$\tilde{a} = a\sqrt{f} \qquad d\tilde{t} = \sqrt{f}\,dt \qquad \tilde{H} = \frac{1}{\sqrt{f}}\left(H + \frac{\dot{f}}{2f}\right)$$

$$\tilde{R} = R + \frac{\delta f}{2f} \qquad \tilde{A} = A + \frac{\delta f}{2f} \qquad \tilde{B} = B \qquad \tilde{E} = E \qquad \delta\tilde{\phi} = \delta\phi\sqrt{\frac{3}{2}\left(\frac{f'}{f}\right)^2 + \frac{\omega}{f}}$$

Curvature perturbation on comoving hypersurface is frame-independent.

$$\tilde{R}_c = \tilde{R} - \frac{\tilde{H}}{d\tilde{\phi}/d\tilde{t}}\delta\tilde{\phi} = R - \frac{H}{d\phi/dt}\delta\phi = R_c$$

Makino & Sasaki (86)
Hwang & Noh (96)
Tsujikawa & Gumjudpai (04)

- $\tilde{R}_c \left(= R_c\right)$ is conserved on superhorizon scales and observed on the sky.

- Nonlinear equivalence has been shown recently

Chiba and Yamaguchi (08), Gong, Hwang, Park, Sasaki and Song (11)

- Tensor perturbation is also conformal invariant.

Multi-field inflation

e.g., Sasaki &Taaka (98) , Gordon, et.al (00)

compactifications in string theory (moduli)

$$-\frac{1}{2}G_{IJ}(\phi)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J - V(\phi)$$

→ Curvature perturbation is not conserved on superhorizon scales, because of the isocurvature mode.

$$\zeta \equiv \mathcal{R} - \frac{H}{\dot{\rho}}\delta\rho.$$

⟺ $\mathcal{R}_c \approx \zeta$ on superhorizon scales

$$\dot{\zeta} \approx -\frac{H}{\rho+p}\delta p_{\rm nad}$$

$$\delta p_{\rm nad} = \delta p - \frac{\dot{p}}{\dot{\rho}}\delta\rho.$$

$$\delta p_{\rm nad} \approx -\frac{2G_{IJ}\dot{\phi}^I}{G_{LM}\dot{\phi}^L\dot{\phi}^M}G_{KN}\frac{D\dot{\phi}^N}{dt}\mathcal{K}^{JK}$$

$$\mathcal{K}^{JK} := \delta\phi^J\dot{\phi}^K - \delta\phi^K\dot{\phi}^J$$

entropy perturbations

Curvature perturbation is conserved

- for a single-field inflation $\mathcal{K}^{\phi\phi} = 0$
- for a geodesic in the curved manifold $\frac{D\dot{\phi}^N}{dt} = 0$

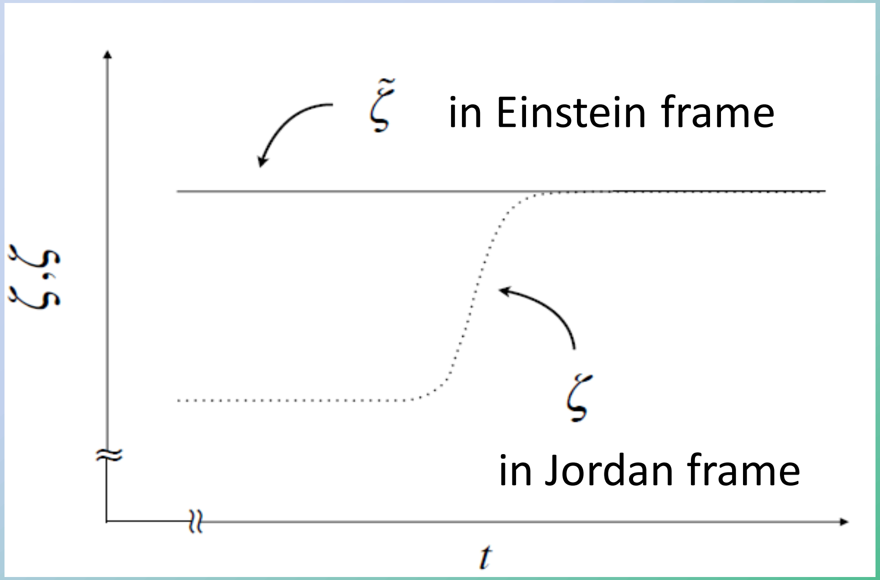
e.g., straight trajectory on a flat field manifold

- If we add nonminimal coupling of multi scalar fields to gravuty, $f(\phi)R$

evolutions of curvature perturbation becime *frame-dependent.*

Even though the background evolution is the same in both frames,
different between frames appear at the perturbation level.

After formulating the evolutions of the curvature perturbation in both frames,
we will construct a model that curvature perturbations behave differently
between frames though the background evolutions are the same.



Multi-field inflation with nonminimal coupling

We discuss the evolution of the cosmological perturbations in the multi-field inflation model with nonminimal coupling.

$$S=\int d^4x\sqrt{-g}\left\{f(\phi)R-\frac{1}{2}G_{IJ}(\phi)g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J-V(\phi)\right\}$$

$I,\ J=1,...,N$ N scalar fields

$f(\phi)$ Nonminimal coupling function

$V(\phi)$ Potential

$G_{IJ}(\phi)$ Field space metric

Moving to the Einstein frame... Kaiser (10)

$g_{\mu\nu} = \Omega \tilde{g}_{\mu\nu}$ $\Omega = \frac{1}{2\kappa^2 f}$

$$S = \int d^4x \sqrt{-\tilde{g}} \left\{ \frac{\tilde{R}}{2\kappa^2} - \frac{1}{2} S_{IJ} \tilde{g}^{\mu\nu} \tilde{\nabla}_\mu \phi^I \tilde{\nabla}_\nu \phi^J - \frac{1}{(2\kappa^2 f)^2} V \right\}$$

$$S_{IJ} = \frac{1}{2\kappa^2 f} \left[G_{IJ} + 3 \frac{f_I f_J}{f} \right]$$

$$\tilde{V} = V / (2\kappa^2 f)^2$$

- The field space metric in the Einstein frame is different from that in the Jordan frame.
- Even if in Jordan frame the field space is diagonal, $G_{IJ} \propto \delta_{IJ}$, in general

that in Einstein frame S_{IJ} is non-diagonal,
except for the case that at most only one $f_I \neq 0$.

Jordan frame analysis

1. Background equations

FLRW Metric

$$ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j \qquad i, j = 1,\ 2,\ 3.$$

Energy-momentum of the matter

$$T_{\mu\nu} = p g_{\mu\nu} + (\rho + p) u_\mu u_\nu = \begin{pmatrix} \rho & 0 \\ 0 & a^2 p\, \delta_{ij} \end{pmatrix}$$

2. Perturbations following the notation of Kodama and Sasaki (84)

Metric perturbations

$$ds^2 = -(1 + 2AY) dt^2 - 2aBY_i dt dx^i + a^2 \left[(1 + 2\mathcal{R})\, \delta_{ij} + 2H_T \frac{1}{k^2} Y_{,ij} \right] dx^i dx^j$$

Matter perturbations

$$\delta T_{00} = -\rho \delta g_{00} + \delta\rho Y,$$
$$\delta T_{0i} = \delta T_{i0} = p \delta g_{0i} - \delta q Y_{,i} \quad \text{and}$$
$$\delta T_{ij} = \delta T_{ji} = p \delta g_{ij} + a^2 \left(\delta p Y \delta_{ij} + p \Pi_T Y_{ij} \right).$$

3. Curvature perturbations on large scales

Curvature perturbations on the uniform density hypersurface

$$\zeta \equiv \mathcal{R} - \frac{H}{\dot{\rho}}\delta\rho$$

Large scale limit $k \ll aH$

$$\dot{\zeta} \approx -\frac{H}{\rho + p}\delta p_{\rm nad}$$

The nonadiabatic pressure is determined later.

4.Derivative along the background trajectory $\phi^I(t)$

$$DX^I = d\phi^J \nabla_J X^I = dX^I + \Gamma^I_{JK} d\phi^J X^K$$

Vector Covariant derivative

↑

G_{IJ}

5. Gauge invariant combinations

$$\mathcal{K}^{JK} := \delta\phi^J \dot{\phi}^K - \delta\phi^K \dot{\phi}^J$$

$$\mathcal{H}^{IJ} := \ddot{\phi}^J \delta\phi^I - \dot{\phi}^I \dot{\delta\phi}^J + A\dot{\phi}^I \dot{\phi}^J$$

entropy perturbation

$$\mathcal{H}^{\phi\phi} = 0$$

$$\left(G_{IJ} + \frac{3f_I f_J}{f}\right)\mathcal{H}^{IJ} + G_{IJ,K}\dot{\phi}^I \mathcal{K}^{JK} \approx \frac{3f_K}{2f}(G_{IJ} + 2f_{IJ})\dot{\phi}^I \mathcal{K}^{JK}$$

hold on superhorizon scales

5. Nonadiabatic pressure is given by the combinations of

$$\delta p_{\rm nad} = \mathcal{N}_{IJ}\mathcal{K}^{IJ} + \mathcal{P}_{IJ}\dot{\mathcal{K}}^{IJ} + \mathcal{Q}_{IJ}\mathcal{H}^{IJ} + \mathcal{T}_{IJ}\dot{\mathcal{H}}^{IJ}$$

$$\mathcal{N}_{JK} = \frac{1}{2\kappa^2 f}\left\{\frac{2V_K\left((G_{IJ}+2f_{IJ})\dot\phi^I - 2Hf_J\right)}{2\kappa^2 f(\rho+p)} + (5+3\mathcal{S})H\frac{2f_K}{\dot f}f_{IJ}\dot\phi^I - \frac{3f_K}{f}f_{IJ}\dot\phi^I \right.$$
$$\left. + \frac{2f}{3\dot f}\frac{d}{dt}\left(\frac{3f_K}{f}f_{IJ}\dot\phi^I\right) - \frac{f_K}{f}G_{IJ}\dot\phi^I\right\},$$

$$\mathcal{P}_{JK} = \frac{1}{2\kappa^2 f}\left\{\frac{2f_K}{\dot f}f_{IJ}\dot\phi^I\right\},$$

$$\mathcal{Q}_{JK} = \frac{1}{2\kappa^2 f}\left\{\frac{3f_Jf_K}{f} - \frac{4V_Jf_K}{2\kappa^2 f(\rho+p)} - (5+3\mathcal{S})\frac{2f_Jf_K}{\dot f} - \frac{2f}{3\dot f}\frac{d}{dt}\left(\frac{3f_Jf_K}{f}\right)\right\},$$

$$\mathcal{T}_{JK} = -\frac{1}{2\kappa^2 f}\left\{\frac{2f_Jf_K}{\dot f}\right\}.$$

$$\mathcal{S} = \frac{\dot f}{2Hf}\left(1+\frac{4p}{3(\rho+p)}\right) + \frac{1}{6H\dot H f}\left(\dddot f + 5H\ddot f\right),$$

$$\rho = \frac{1}{2\kappa^2 f}\left[\frac{1}{2}G_{IJ}\dot\phi^I\dot\phi^J + V - 6H\dot f\right],$$
$$p = \frac{1}{2\kappa^2 f}\left[\frac{1}{2}G_{IJ}\dot\phi^I\dot\phi^J - V + 2\ddot f + 4H\dot f\right],$$

Jordan frame analysis is in general involved for an actual use

- for a single-field model, $\mathcal{K}^{\phi\phi} = 0$ and $\mathcal{H}^{\phi\phi} = 0$, curvature perturbation is conserved

- for the minimally coupling $f = \text{const.}$,

$$\delta p_{\text{nad}} \approx -\frac{2G_{IJ}\dot{\phi}^I}{G_{LM}\dot{\phi}^L\dot{\phi}^M} G_{KN} \frac{D\dot{\phi}^N}{dt} \mathcal{K}^{JK}$$

The well-know results are recovered.

Einstein frame analysis

1. Curvature perturbations

Non-adiabatic pressure for the evolution of $\tilde{\zeta}$

$$\delta\tilde{p}_{nad} = -\frac{1}{(2\kappa^2 f)^2}\left\{\frac{2V_I\phi'^I}{3\tilde{H}(\tilde{\rho}+\tilde{p})}\delta\tilde{\rho}_m + 2V_I\tilde{\Delta}^I - \frac{4V f'\delta\tilde{\rho}}{3\tilde{H}f(\tilde{\rho}+\tilde{p})} - \frac{4V\delta f}{f}\right\}$$

$$\delta\tilde{\rho}_m := \delta\tilde{\rho} - 3\tilde{H}\delta\tilde{q}$$

$$\longrightarrow \quad \delta\tilde{p}_{nad} = -\frac{2S_{IJ}\phi'^I}{(2\kappa^2 f)^2(\tilde{\rho}+\tilde{p})}S_{KL}\frac{D^{(S)}\phi'^L}{d\tilde{t}}\tilde{\mathcal{K}}^{JK}$$

Covariant derivative
in the Einstein frame

Entropy perturbations follow the trivial conformal rescaling $\tilde{\mathcal{K}}^{IJ} = \mathcal{K}^{IJ}/\sqrt{2\kappa^2 f}$

Adiabaticity is conformally irrelevant.

For the geodesic in the field space of the Einstein frame, the curvature perturbation is conserved. $\frac{D^{(S)}\phi'^L}{d\tilde{t}} = 0$

Comparing frames

$$\zeta - \tilde{\zeta} \approx \mathcal{A}_{JK}\mathcal{K}^{JK} + \mathcal{B}_{JK}\dot{\mathcal{K}}^{JK}$$

$$\mathcal{A}_{JK} = \frac{1}{\mathcal{C}}\Bigg\{\Bigg[\Bigg(\frac{G_{PQ}\dot{\phi}^P\dot{\phi}^Q + 2\left(\ddot{f} - H\dot{f}\right)}{2f} - 2H^2\Bigg) f_K G_{IJ} + 2H\dot{\phi}^L f_{KL} G_{IJ} - 2H\dot{f} G_{IJ,K}\Bigg]\dot{\phi}^I - 2H f_K G_{IJ}\ddot{\phi}^I\Bigg\}$$

$$\mathcal{B}_{JK} = \frac{2H f_K G_{IJ}\dot{\phi}^I}{\mathcal{C}} \quad \text{and}$$

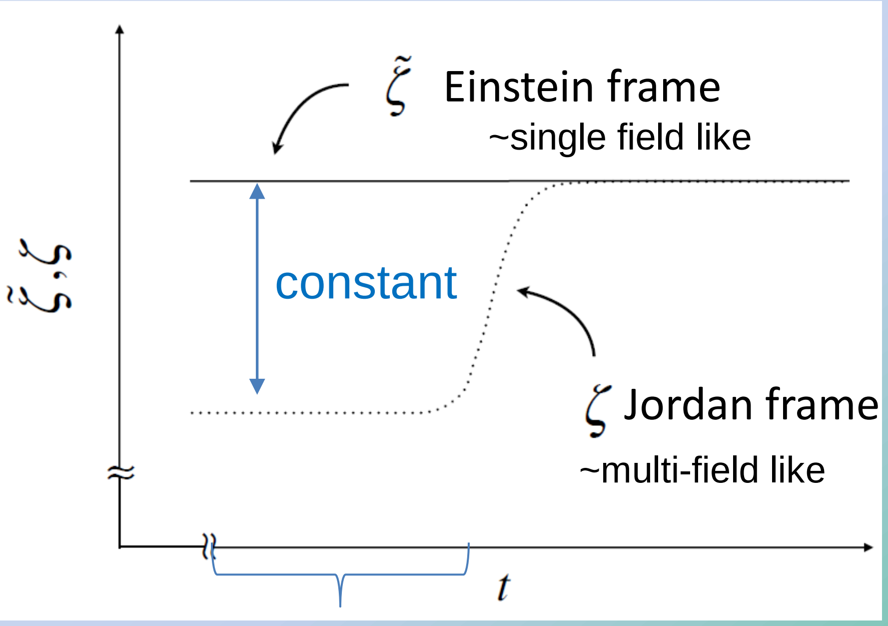
$$\mathcal{C} = 2\kappa^2 f S_{MN}\dot{\phi}^M\dot{\phi}^N\left(G_{PQ}\dot{\phi}^P\dot{\phi}^Q + 2\left(\ddot{f} - H\dot{f}\right)\right).$$

The difference between frames is written in terms of the entropy mode

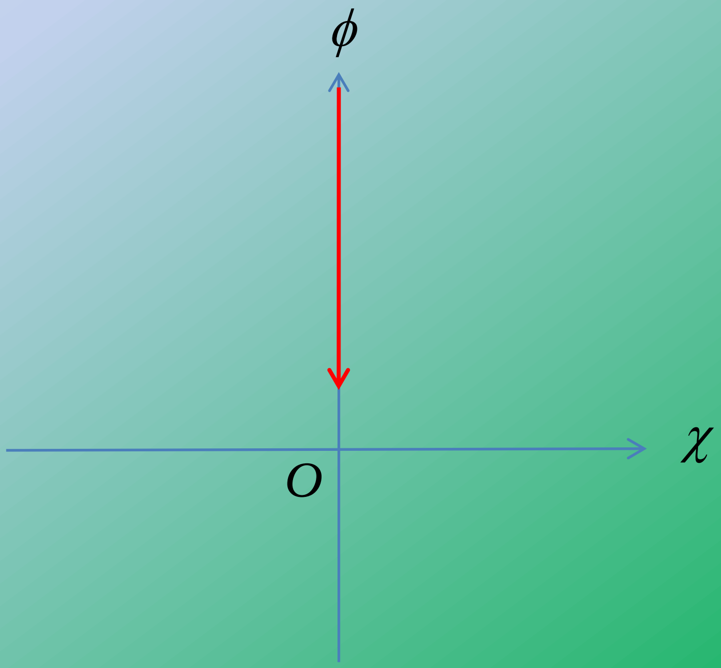
- Adiabatic case: $\delta\phi^I \propto \dot{\phi}^I$
  the field evolution is effectively single-field like and the curvature perturbation is equivalent. $\dot{\tilde{\zeta}} = \dot{\zeta} = 0 \text{ and } \tilde{\zeta} = \zeta.$
- ζ itself is not an observable.
  Observables themselves are conformally invariant irrespective of whether the adiabatic limit is reached.

Two-field example

We will construct a two-field example in which during inflation curvature perturbations are conserved with different amplitude



Inflation



Both fields finally decay into radiation,
and the subsequent evolution of the universe becomes adiabatic,
the difference in perturbations eventually disappear.

$$S = \int d^4x\sqrt{-g}\left\{f(\phi,\chi)R - \frac{1}{2}G_{\phi\phi}(\partial\phi)^2 - \frac{1}{2}G_{\chi\chi}(\partial\chi)^2 - G_{\phi\chi}g^{\mu\nu}\partial_\mu\phi\partial_\nu\chi - V(\phi,\chi)\right\}$$

- $\dot{\chi} = 0$ during inflation.
- Nonadiabatic pressure in the Einstein frame

$$\delta\tilde{p}_{nad} = \frac{2\tilde{\mathcal{K}}^{\phi\chi}\phi'^3}{2f(2f)^2(\tilde{\rho}+\tilde{p})}\{(fG_{\chi\phi}+3f_\phi f_\chi)G_{\phi\phi,\phi} - (fG_{\phi\phi}+3f_\phi^2)(2G_{\chi\phi,\phi} - G_{\phi\phi,\chi}) + (G_{\phi\phi}+6f_{\phi\phi})(f_\phi G_{\phi\chi} - f_\chi G_{\phi\phi})\},$$

Conserved curvature perturbation in Einstein frame : $\delta\tilde{p}_{\rm nad} = 0$

- Difference of curvature perturbations

$$\zeta - \tilde{\zeta} = (\mathcal{A}_{\phi\chi} - \mathcal{A}_{\chi\phi})\mathcal{K}^{\phi\chi} + (\mathcal{B}_{\phi\chi} - \mathcal{B}_{\chi\phi})\dot{\mathcal{K}}^{\phi\chi}. \qquad \mathcal{K}^{\phi\chi} = -\dot{\phi}\delta\chi$$

For simplicity, $S_{\phi\chi} = S_{\chi\phi} = 0$

$$G_{\phi\chi} = G_{\chi\phi} = 0, \text{ and either } f_\phi = 0 \text{ or } f_\chi = 0.$$

We choose $f = f(\chi)$

- During inflation $f(\chi = 0) = m_p^2$

The background evolutions are the same.

→ Difference between frames appears in perturbations $\delta f = f_\chi \delta\chi \propto K_{\phi\chi}$

To obtain $\delta\tilde{p}_{\rm nad} = 0$, we further impo $fG_{\phi\phi,\chi} = G_{\phi\phi}f_{\chi}$

- Equations for the background in both frame

$$f_\chi R = V_\chi \qquad \frac{D\dot\phi}{dt} + 3H\dot\phi + \frac{V_\phi}{G_{\phi\phi}} = 0$$

$$3H^2 = \frac{G_{\phi\phi}\dot\phi^2}{4f} + \frac{V}{2f} \qquad 2\dot H = -\frac{1}{2f}G_{\phi\phi}\dot\phi^2$$

Equations of motion for perturbations

$$\ddot{\delta\chi} + (3H + c_\chi)\dot{\delta\chi} + \left(\frac{k^2}{a^2} - m_\chi^2\right)\delta\chi = 0$$

- Slow-roll approximations

$$\eta = \frac{1}{H\dot\phi}\frac{D\dot\phi}{dt} \qquad \epsilon = -\frac{\dot H}{H^2} \qquad \text{and} \qquad \xi = \frac{\ddot H}{H\dot H}$$

$$\epsilon \ll 1 \quad \Rightarrow \quad \frac{f}{G_{\phi\phi}}\left(\frac{V_\phi}{V}\right)^2 \ll 1$$

$$\eta,\ \xi \ll 1 \quad \Rightarrow \quad \frac{f}{G_{\phi\phi}}\frac{V_{\phi\phi}}{V} \ll 1$$

Difference of curvature perturbation in slow-roll appoximations

$$\frac{1}{H}\frac{\mathrm{d}}{\mathrm{dt}}\ln(\zeta-\tilde{\zeta}) = -\frac{\ddot{\delta\chi} - H(1+2\eta)\dot{\delta\chi} + 2H^2(\eta+\epsilon)\delta\chi}{(H^2+\dot{H})\delta\chi - H\dot{\delta\chi}}$$

Now we would like to obtain $\frac{1}{H}\frac{\mathrm{d}}{\mathrm{dt}}\ln(\zeta-\tilde{\zeta}) \sim O(\epsilon)$

Supposing $m_\chi^2/H^2 \sim O(\epsilon)$ and setting $G_{\chi\chi} = 1$, the solution for fluctuation is given by

$$\delta\chi \simeq const = \frac{H}{\sqrt{2k^3}}(1+3f_\chi^2/f)^{-1/2}$$

If $f_\chi/\sqrt{f} \sim O(\epsilon^{1/2})$, we can realize

$$\frac{1}{H}\frac{\mathrm{d}}{\mathrm{dt}}\ln(\zeta-\tilde{\zeta}) \sim O(\epsilon)$$

$$\zeta-\tilde{\zeta} = \frac{f_\chi}{2f\epsilon}\frac{H}{\sqrt{2k^3}}(1+3f_\chi^2/f)^{-1/2}$$

reproducing the correct amplitude

$$\frac{f_\chi H}{f\varepsilon} \sim \frac{H}{m_p\varepsilon^{1/2}} \sim 10^{-5}$$

- Condition for $m_\chi^2/H^2 \sim O(\epsilon)$

$$m_\chi^2 = \frac{8f_\chi^2 f H^2(3-\epsilon) + 12 f_{\chi\chi} f^2 H^2 (2-\epsilon) - 2f^2 V_{\chi\chi} + f^2 G_{\phi\phi,\chi\chi}\dot{\phi}^2}{2f(3f_\chi^2 + f)}$$

Assuming

$$V = a(\phi)\chi + b(\phi)\chi^2 + V_0(\phi), \qquad f = \frac{1}{2}e^{c\chi}, \qquad \text{and} \qquad G_{\phi\phi} = e^{\phi + c\chi}$$

which satisfies $fG_{\phi\phi,\chi} = G_{\phi\phi} f_\chi$ and $f_\chi R = V_\chi$ so long as $a(\phi) = cR/2$

$$\Rightarrow \quad m_\chi^2 = \frac{4}{2+3c^2}\left\{c^2 H^2 (6-2\epsilon) - b(\phi)\right\}$$

To achieve $m_\chi^2/H^2 \sim O(\epsilon)$ if $b(\phi) = c^2 H^2 (6-2\epsilon) \pm O(\epsilon)$

or $c^2 \sim O(\epsilon)$ $b(\phi) \sim O(\epsilon)$

- Dynamics of inflaton

$$V_0 = m^2\phi^2 \qquad \phi = \ln(e^{\phi_i} - 2\sqrt{m^2/3}t)$$

e-foldings $N \simeq \sqrt{\frac{m^2}{3}}\phi_i t \simeq \sqrt{\frac{m^2}{3}}\frac{\phi_i}{\alpha}(\phi_i - \phi) = \frac{1}{2}e^{\phi_i}\phi_i(\phi_i - \phi)$

$$N \gg 60 \ (\text{e.g. if } \phi_i \sim O(10))$$

Generalized model

Expanding around $\chi = 0$

$$V = \sum_n V_{(n)}(\phi)\chi^n, \qquad f = \sum_n f_{(n)}\chi^n, \qquad \text{and} \qquad G_{\phi\phi} = \sum_n G^{(n)}_{\phi\phi}(\phi)\chi^n$$

$$\frac{m_\chi^2}{H^2} = \frac{2}{\left(1 + \frac{3f_{(1)}^2}{f_{(0)}}\right)} \left\{ 2\frac{f_{(1)}^2}{f_{(0)}}(3-\epsilon) + 6f_{(2)}(2-\epsilon) - \frac{V_{(2)}}{H^2} + \frac{G^{(2)}_{\phi\phi}\dot{\phi}^2}{2H^2} \right\}$$

In order to obtain $m_\chi^2/H^2 \sim O(\epsilon)$

$$\frac{f_{(1)}^2}{f_{(0)}} \sim O(\epsilon), \qquad f_{(2)} \sim O(\epsilon), \qquad \frac{f_{(0)}V_{(2)}}{V_{(0)}} \sim O(\epsilon) \qquad \text{and} \qquad \frac{f_{(0)}G^{(2)}_{\phi\phi}}{G^{(0)}_{\phi\phi}} \sim O(1)$$

→

$$\frac{V_{(1)}}{V_{(0)}} \sim O(\epsilon^{1/2}) \qquad \text{and} \qquad \frac{G^{(1)}_{\phi\phi}}{G^{(0)}_{\phi\phi}} \sim O(\epsilon^{1/2}),$$

with $f_{(0)} = 1/2$

Summary

We have formulated the curvature perturbations in the multi-field inflation with nonminimal coupling.

- We have derived the evolution equations of curvature perturbations in both the Jordan and Einstein frames.

- We then derived the formula relating curvature perturbations on both frames.

  In the multi-field inflation model, the difference may be nonzero and can be written only in terms of the isocurvature perturbations.

  The time evolution can be different, although they become equivalent once the adiabatici limit is reached.

- Based on our formulation, we have constructed an inflationary model in which the amplitude of curvature perturbations are conserved with different amplitudes in different frames.

Thank you