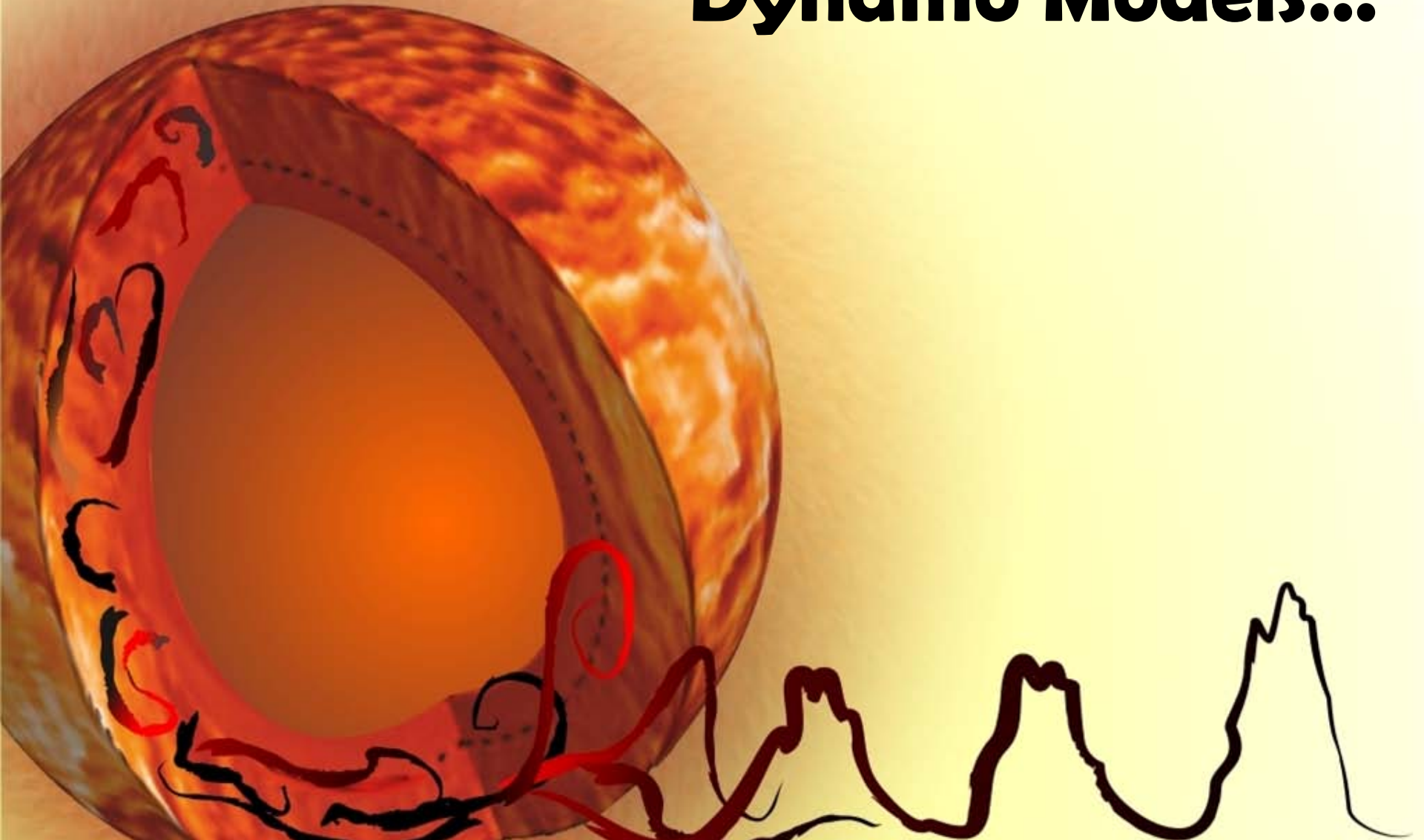


# The next step for Solar Dynamo Models...

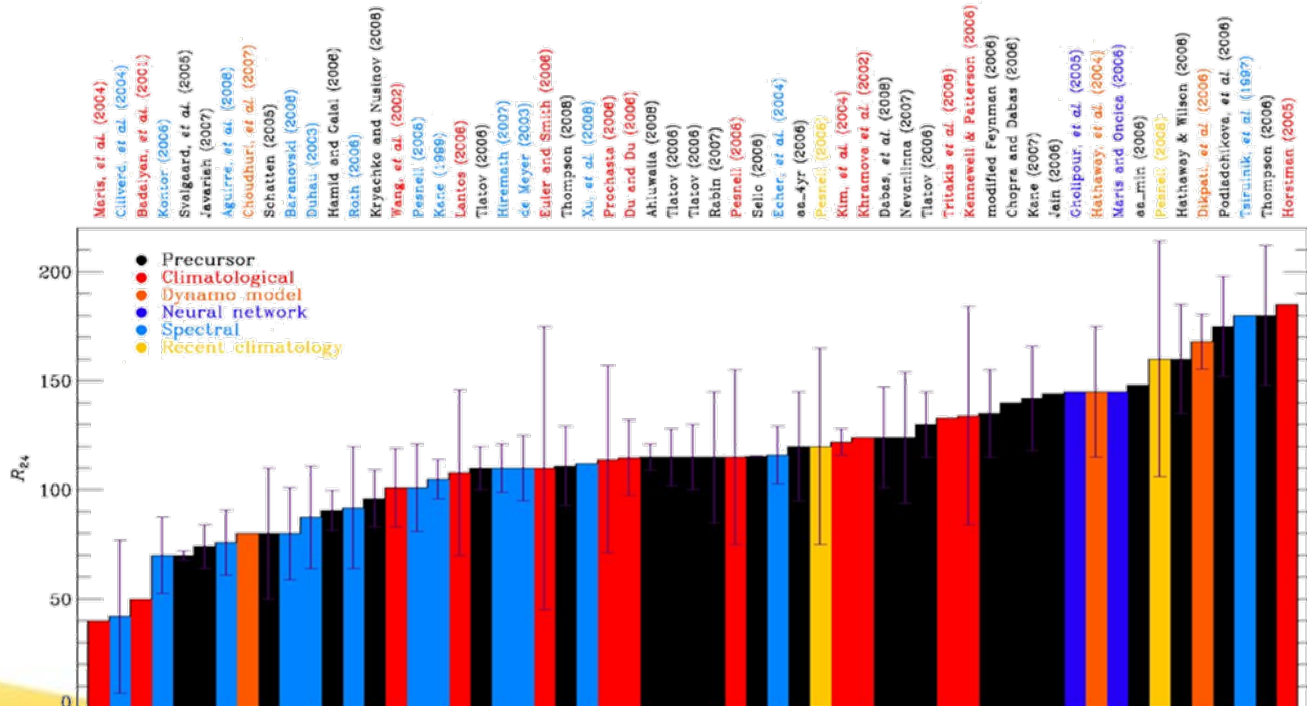


# Space Weather, Solar Cycle and Predictability

- ❑ Space weather conditions many activities:
  - satellite's operation, astronaut activities, power and telecommunications networks, high latitude airplanes and space tourism (in the near future).
- ❑ Solar magnetic activity controls space weather
- ❑ Solar cycle variability is one of the most difficult characteristics to understand/predict
- ❑ Prediction techniques are divided into three categories:
  - precursors, extrapolation (statistical) and model based

Most Promising?

**Solar  
Dynamo  
Models**



# Solar Dynamo Models

**Common to all models:** MHD induction equation, mean field theory (scale separation)

$$\partial \mathbf{B} / \partial t = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Axisymmetry: Large scale fields separated into poloidal and toroidal comp.

Magnetic field

$$\mathbf{B} = B_{\phi} \hat{e}_{\phi} + (\nabla \times A_p \hat{e}_{\phi})$$

Velocity field

$$\mathbf{u} = \Omega \hat{e}_{\phi} + \mathbf{v}_p$$

Small scale properties included in the coefficients of the dynamo equations

Different source terms and locations define the “dynamo zoo”

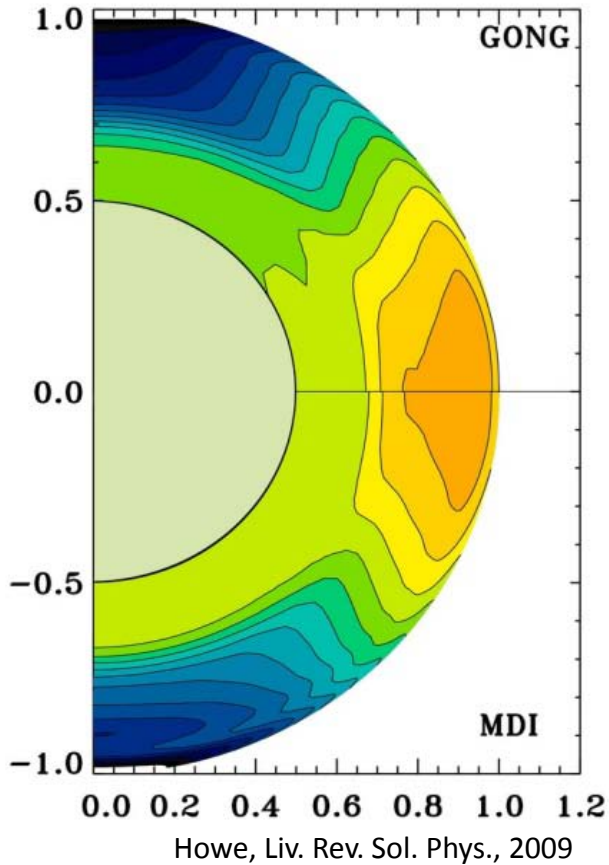
- ❑  $\alpha\Omega$  dynamos - Solar dif. rotation and Coriolis effect
- ❑  $\alpha^2\Omega$  dynamos - Solar dif. Rotation, Coriolis force and turbulence
- ❑ Flux transport dynamos - Solar dif. Rotation, meridional circulation and (usually) active regions decay

**99% of these models run in the kinematic regime, i.e., it's assumed that the flows control the magnetic fields. No back reaction from the field into the flow.**



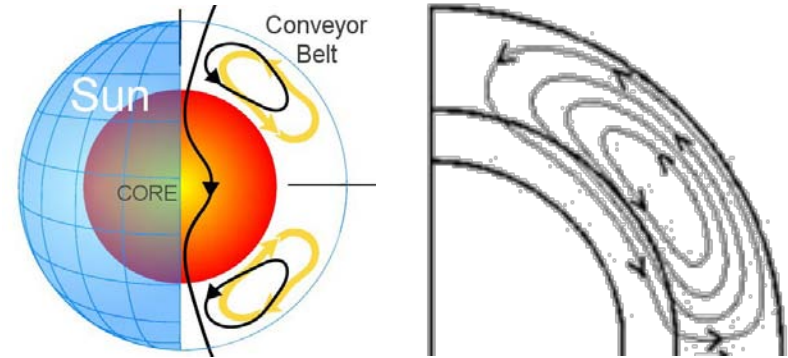
# Large scale plasma velocity fields in the Sun

Differential rotation (strong flow)

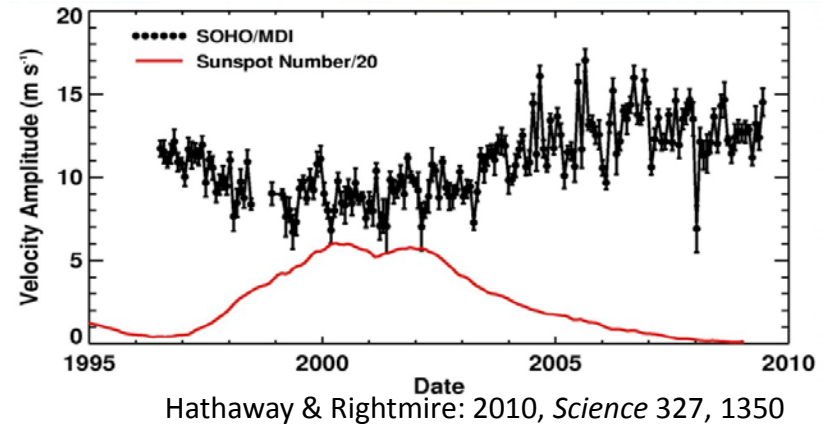


Probed by Helioseismology  
for the entire convection zone

Meridional circulation (weak flow)



Theoretical and observational evidences  
for MC variations from cycle to cycle



Magnetic features tracking (at the surface)

# Modelling a flux transport dynamo

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \quad \text{Induction equation}$$

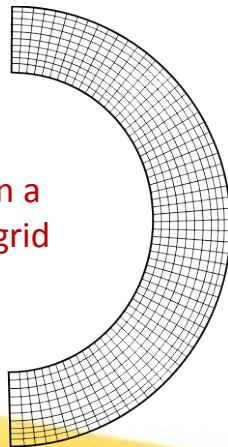
$$\bar{\mathbf{B}} = \mathbf{B}_\phi + \mathbf{B}_p \quad \mathbf{B}_p = \nabla \times (A_p \hat{e}_\phi) \quad \mathbf{u} = \frac{\Omega}{r \sin(\theta)} \hat{e}_\phi + v_p \quad \text{Axisymmetric approximation}$$

Evolution equations for the magnetic field

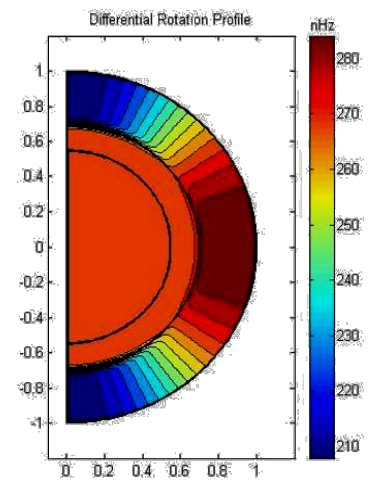
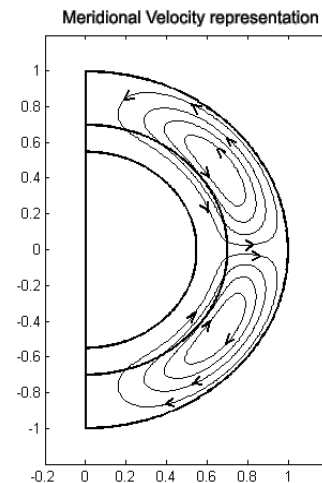
$$\frac{\partial B_\phi}{\partial t} = \eta \left( \nabla^2 - \frac{1}{\bar{r}^2} \right) B_\phi - \bar{r} v_p \cdot \nabla \left( \frac{B_\phi}{\bar{r}} \right) + \frac{1}{\bar{r}} \frac{\partial(\bar{r} B_\phi)}{\partial r} \frac{\partial \eta}{\partial r} - B_\phi \nabla \cdot v_p + \bar{r} [\nabla \times (A_p \hat{e}_\phi)] \cdot \nabla \Omega$$

$$\frac{\partial A_p}{\partial t} = \eta \left( \nabla^2 - \frac{1}{\bar{r}} \right) A_p - \frac{v_p}{\bar{r}} \cdot \nabla (\bar{r} A_p) + \alpha B_\phi$$

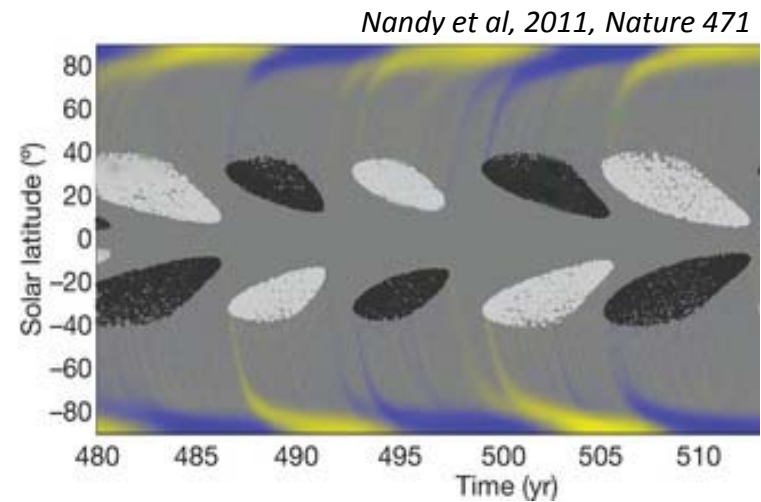
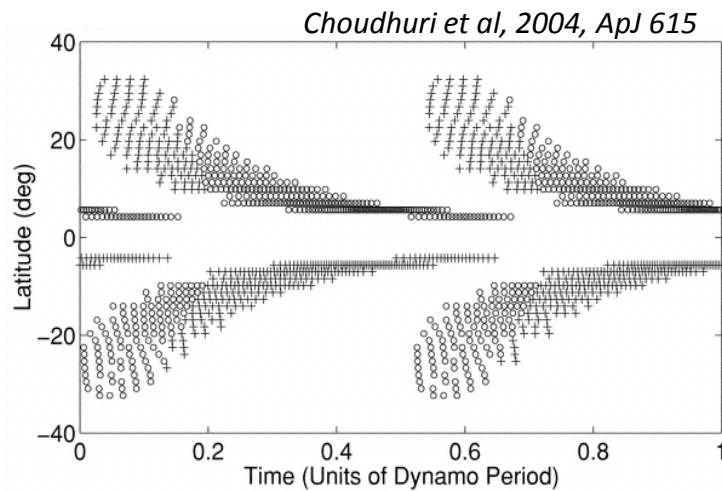
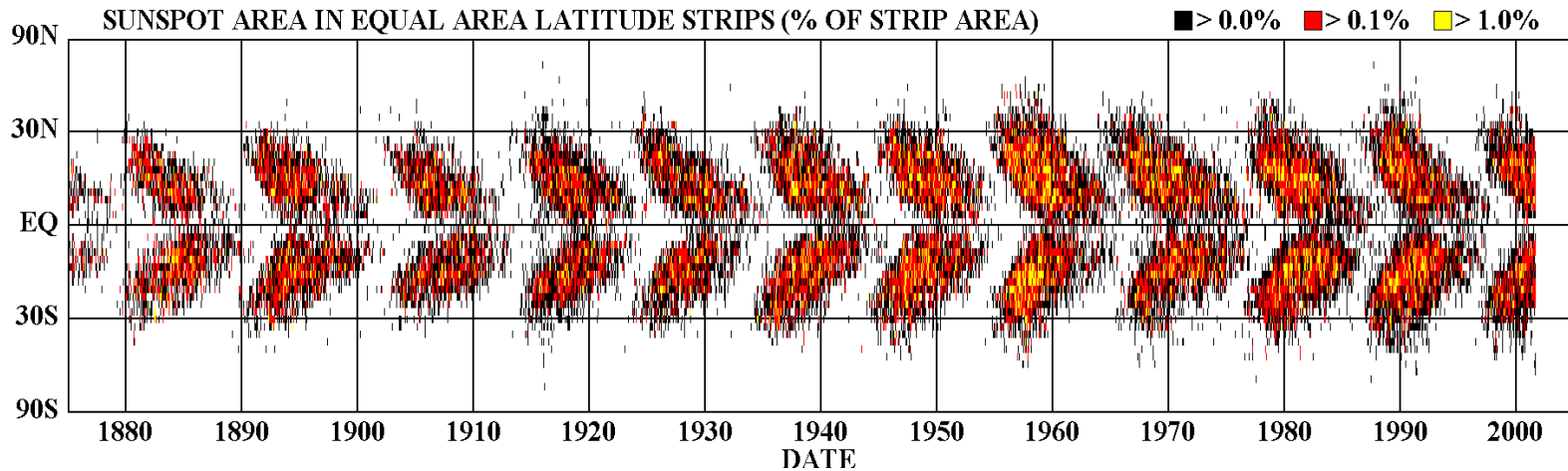
Equations solved in a  
N x M meridional grid



Parameterized  
velocity fields



# Example results from flux transport dynamo models



# Modelling: Global Large-Eddy Simulation of the solar Convection Zone

*Ghizaru, Charbonneau and Smolarkiewicz, ApJL 715, 2010*

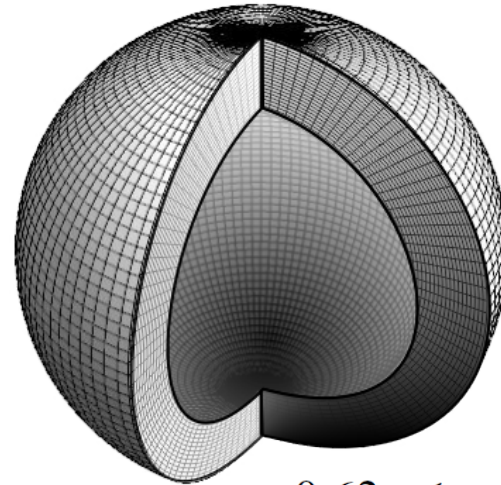
Anelastic form of the ideal MHD equations:

$$\frac{Du}{Dt} = -\nabla\pi' - \mathbf{g}\frac{\Theta'}{\Theta_o} + 2\mathbf{u} \times \boldsymbol{\Omega} + \frac{1}{\mu\rho_o} (\mathbf{B} \cdot \nabla) \mathbf{B},$$

$$\frac{D\Theta'}{Dt} = -\mathbf{u} \cdot \nabla\Theta_e + \mathcal{H} - \alpha\Theta',$$

$$\frac{D\mathbf{B}}{Dt} = (\mathbf{B} \cdot \nabla) \mathbf{u} - \mathbf{B}(\nabla \cdot \mathbf{u}).$$

$$\nabla \cdot (\rho_o \mathbf{u}) = 0, \quad \nabla \cdot \mathbf{B} = 0$$



Grid

$$r = 47$$

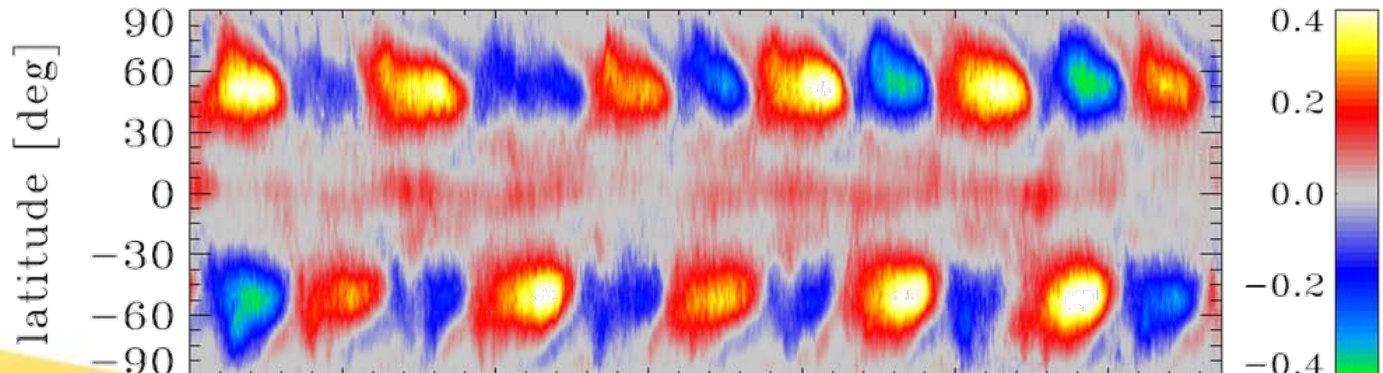
$$\theta = 66$$

$$\phi = 128$$

$$0.62 \leq r/R_{\odot} \leq 0.96$$

**Magnetic Cycles!**

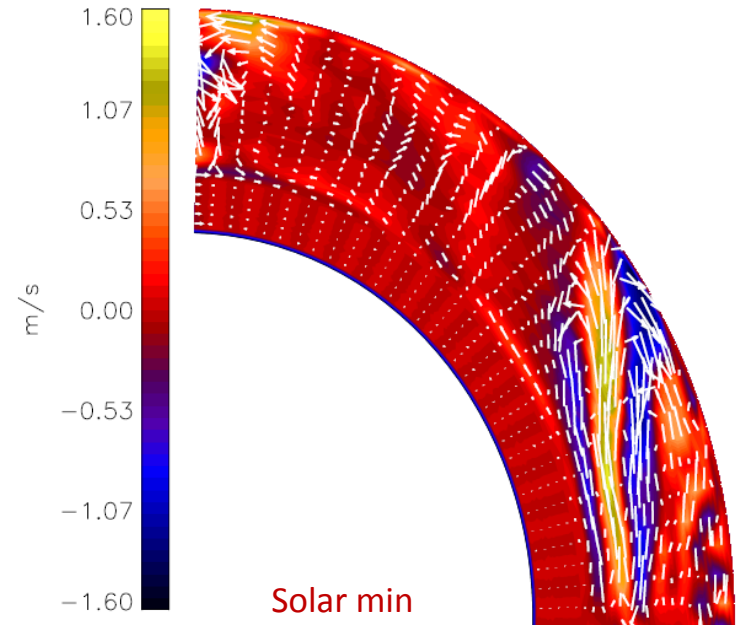
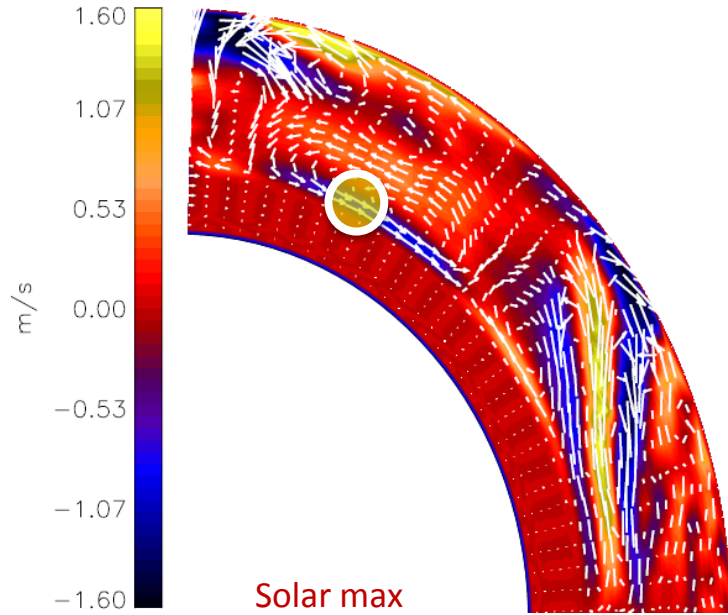
Zonally averaged toroidal magnetic field at the tachocline



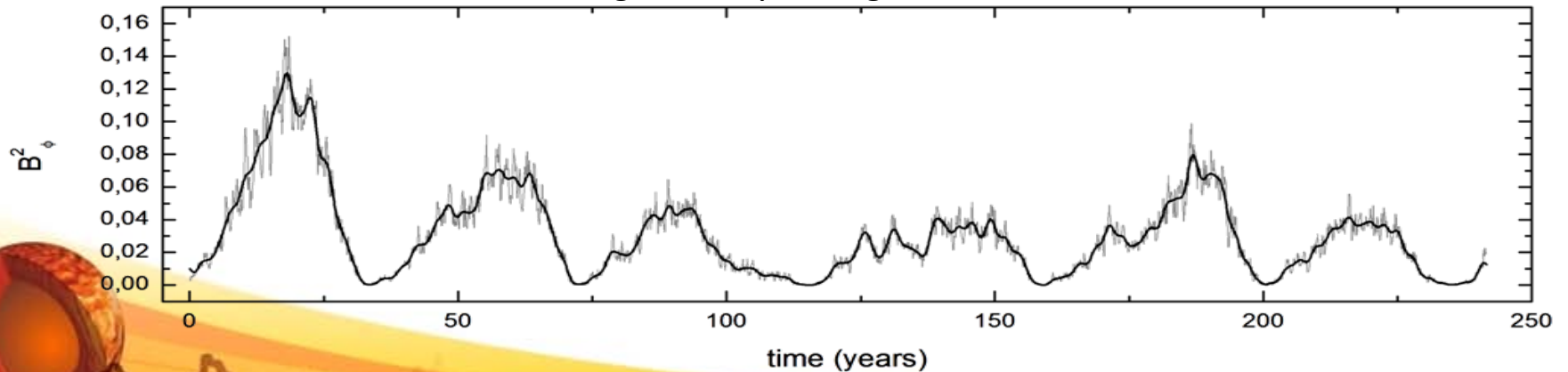
# Analysis of the 3D simulation

## Meridional circulation in the north hemisphere

*Racine et al 2011, Apj 735*



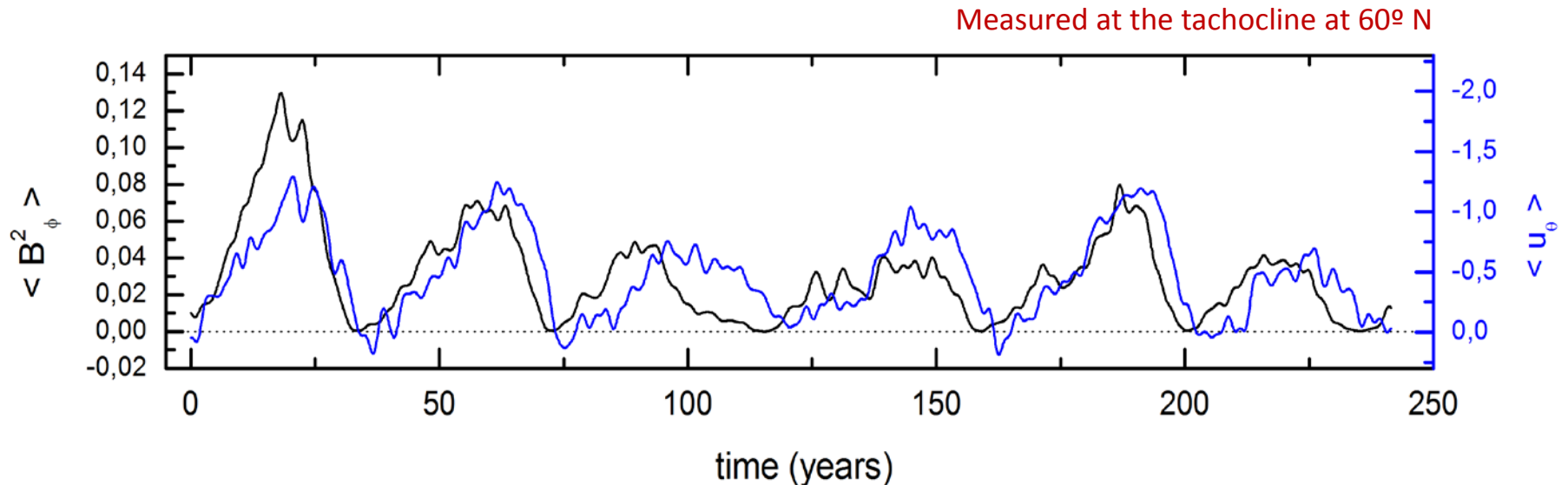
## Longitudinally averaged toroidal field at the tachocline at 60° N





## Analysis of the 3D simulation

Chosen quantities: *Toroidal field,  $B_\phi$  and meridional flow  $u_\theta$  at the tachocline*



- ❑ Meridional flow varies in phase with  $B_\phi$
- ❑ Meridional flow lags behind  $B_\phi$  by 2~3 months

**Kinematic approximation fails!**

**Does this has any influence in the long term evolution of the dynamo?**



# LODM: Low Order Dynamo Model

## Equations for the magnetic field evolution

$$\frac{dB_\phi}{dt} = \left( c_1 - \frac{v_p(t)}{\ell_0} \right) B_\phi + c_2 A_p - c_3 B_\phi^3$$

$$\frac{dA_p}{dt} = \left( c_1 - \frac{v_p(t)}{\ell_0} \right) A_p + \alpha B_\phi$$

## Equations for the meridional flow evolution

$$v_p(t) = v_0 + v(t)$$

$$\frac{dv(t)}{dt} = \underbrace{a B_\phi A_p}_{\text{Lorentz}} - \underbrace{b v(t)}_{\text{Drag}}$$

## Structural coefficients

$$c_1 = \frac{\eta}{\ell_0^2} - \frac{\eta}{R^2} \quad \text{Mag. diffusivity}$$

$$c_2 = \frac{R\Omega}{\ell_0^2} \quad \text{Diff. rotation}$$

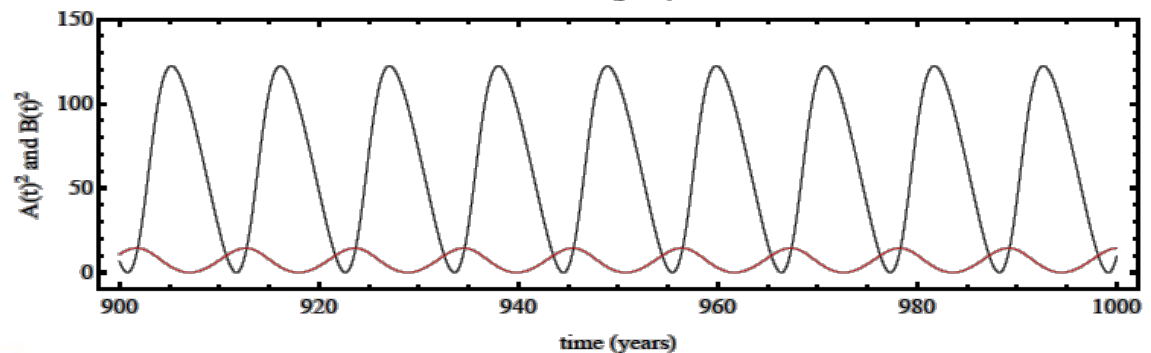
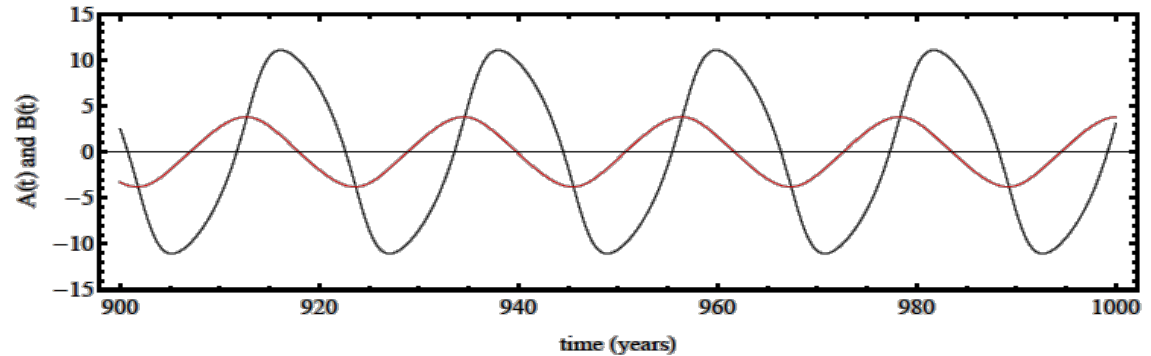
$$c_3 = \frac{\gamma}{8\pi\rho} \quad \text{Mag. buoyancy}$$

## LODM main characteristics:

- Dynamo action
- Polarity change
- Phase shift between **A** and **B**
- Structural coefficients can be “calibrated” from observations

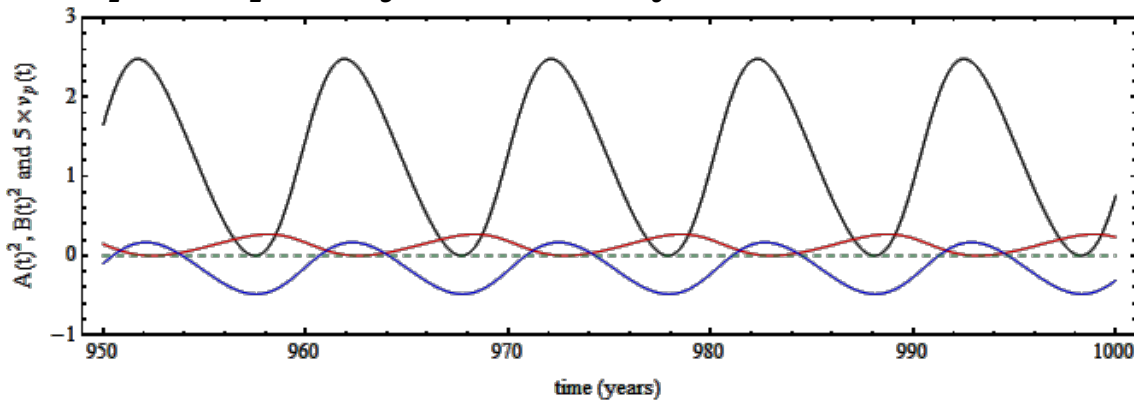
Reference solution without feedback:

$$c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1$$



Solution with feedback:

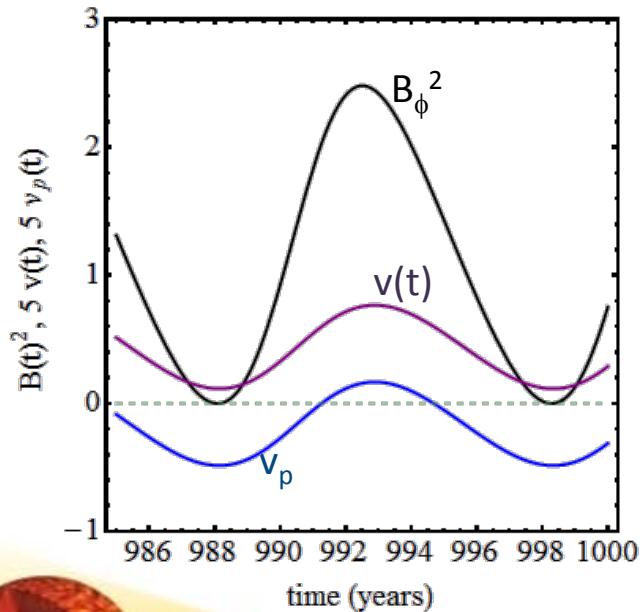
$$c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1, \quad a = 0.1, b = 0.05$$



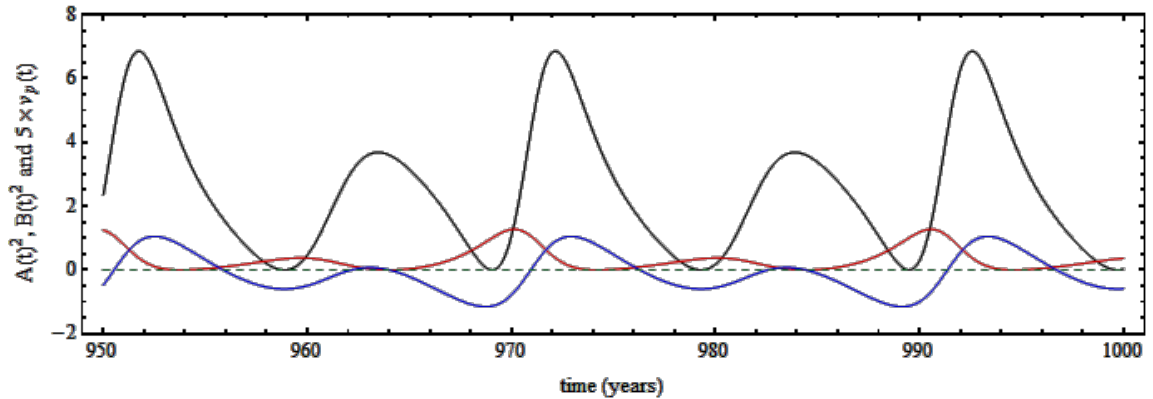
$B_\phi^2$  in black  
 $A_p^2$  in red  
 $v_p$  in blue

$$v_p = v_0 + v(t)$$

*kinetic*  
 +  
*feedback*

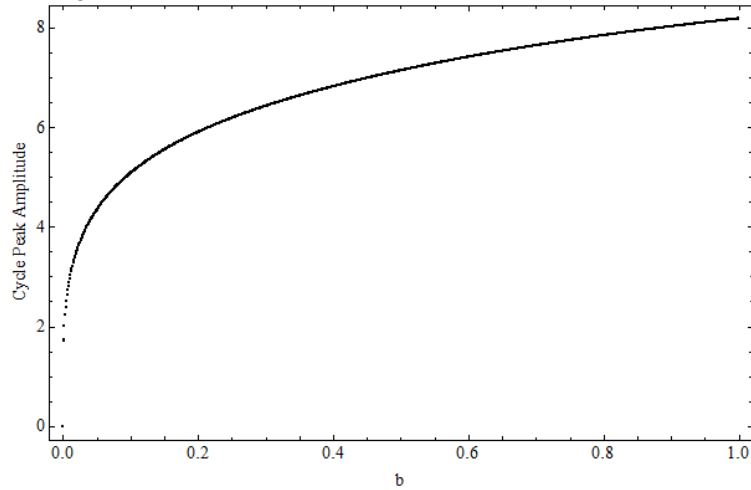


Solution with feedback:  
 $c_1 = -0.01, c_2 = 0.95, c_3 = 0.002, \alpha = -0.1, v_0 = -0.1, \quad a = 0.1, b = 0.25$

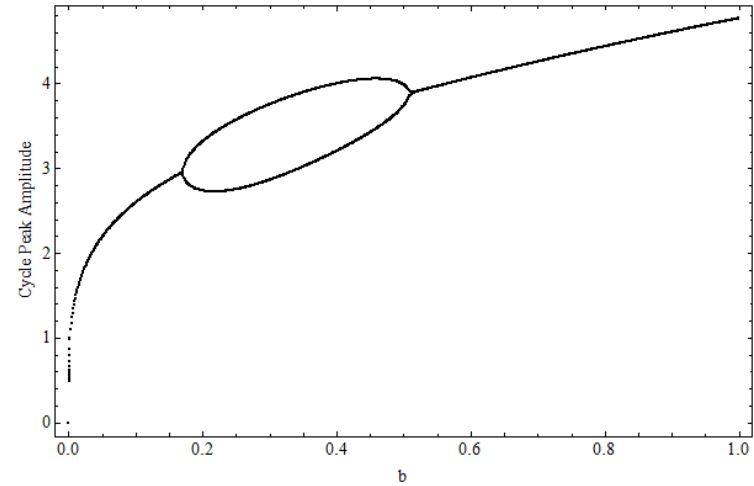


# Bifurcation Maps

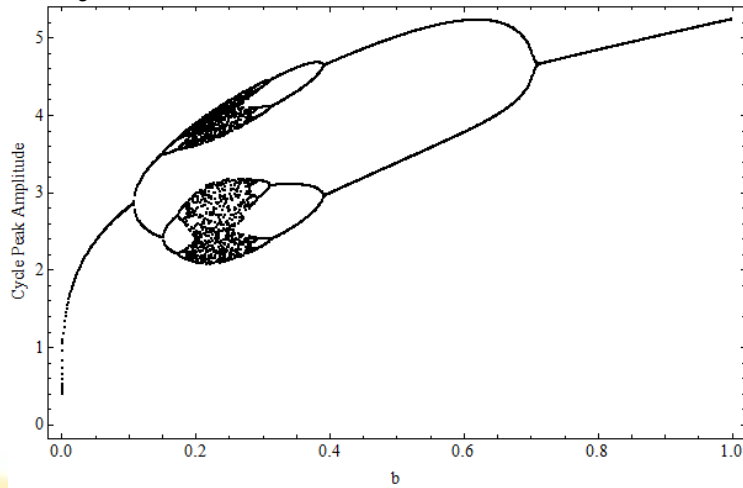
$V_0 = -0.1, a = 0.01$



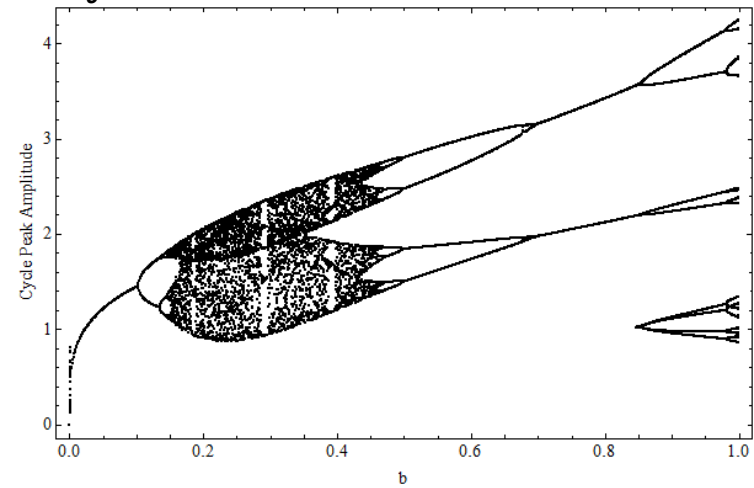
$V_0 = -0.1, a = 0.05$



$V_0 = -0.13, a = 0.05$

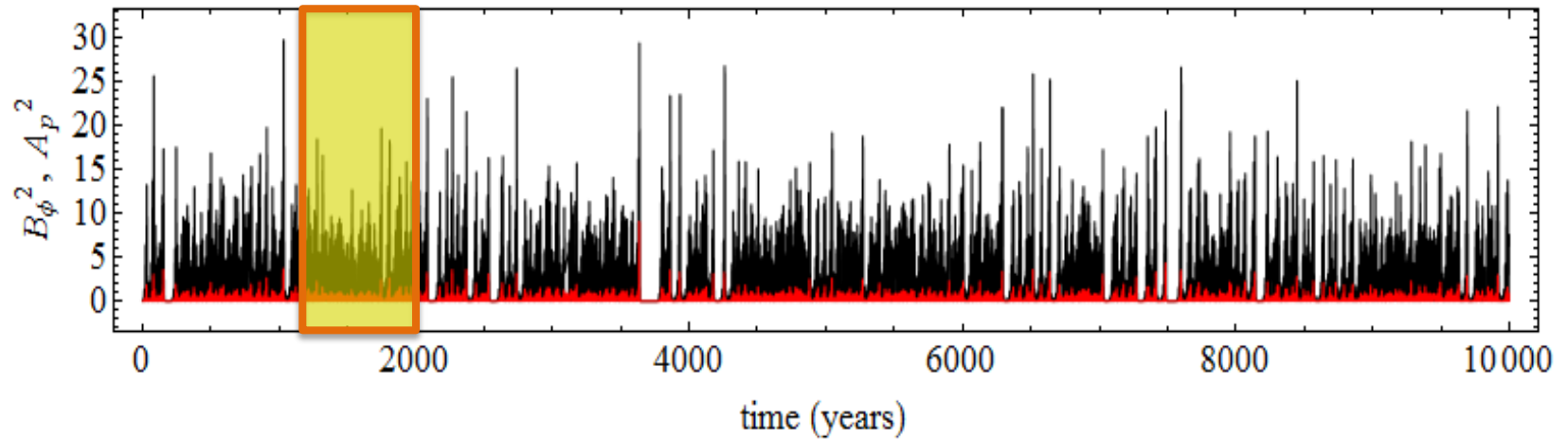


$V_0 = -0.12, a = 0.2$

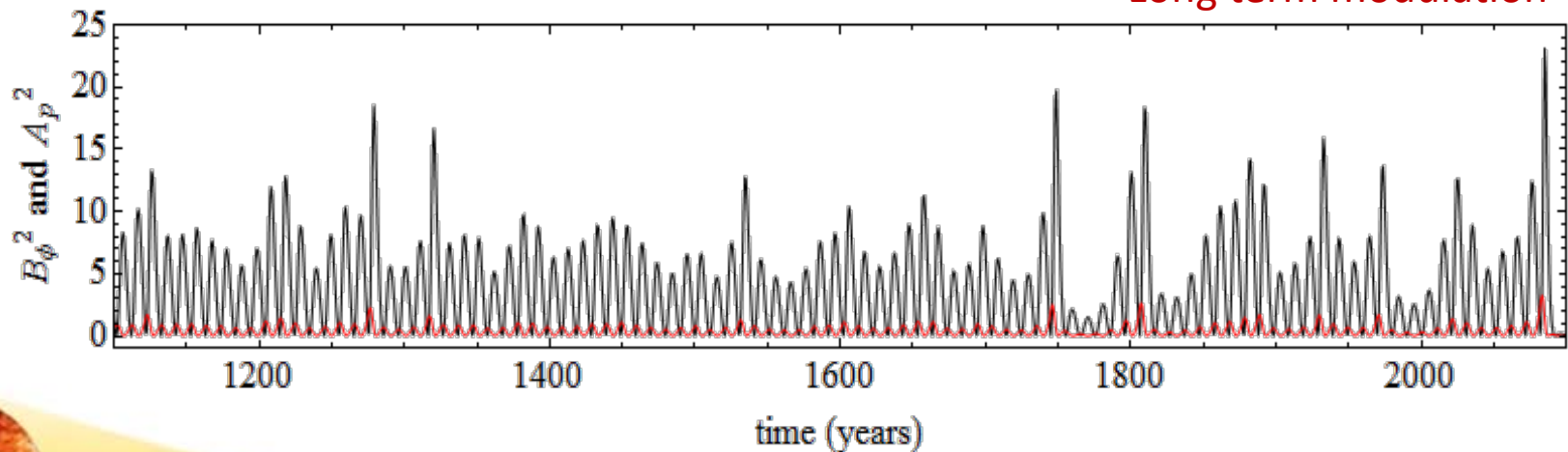


## Stochastic Fluctuations

$\tau = 1$  year,  $\mathbf{a} \in [0.01, 0.04]$ ,  $\mathbf{b} = 0.05$

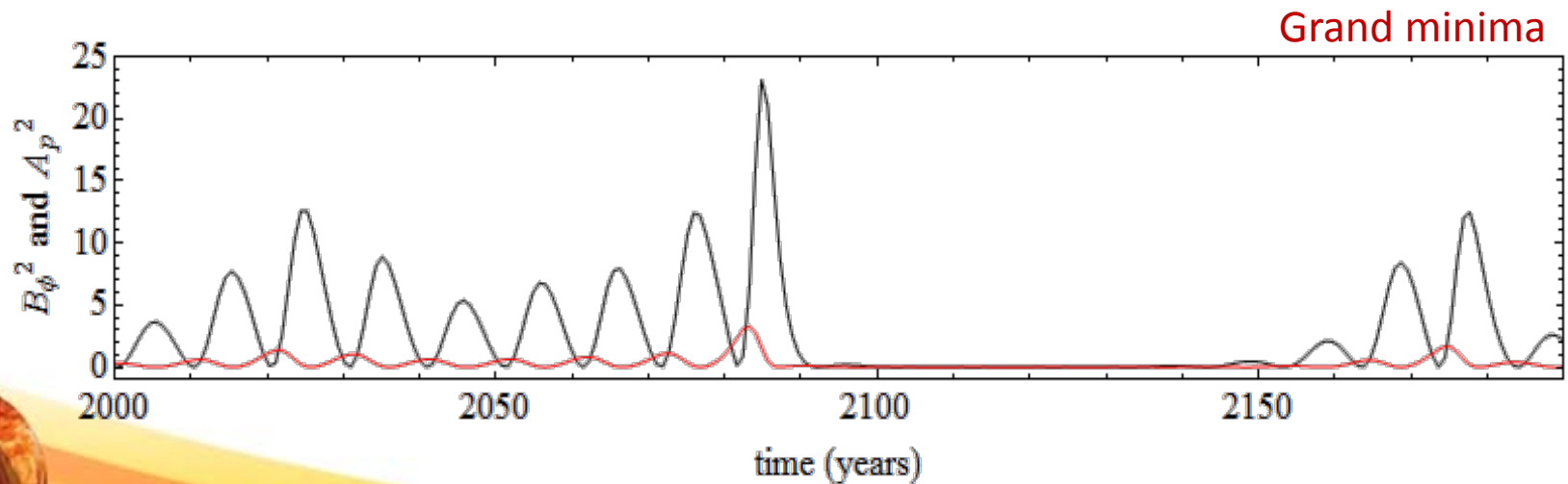
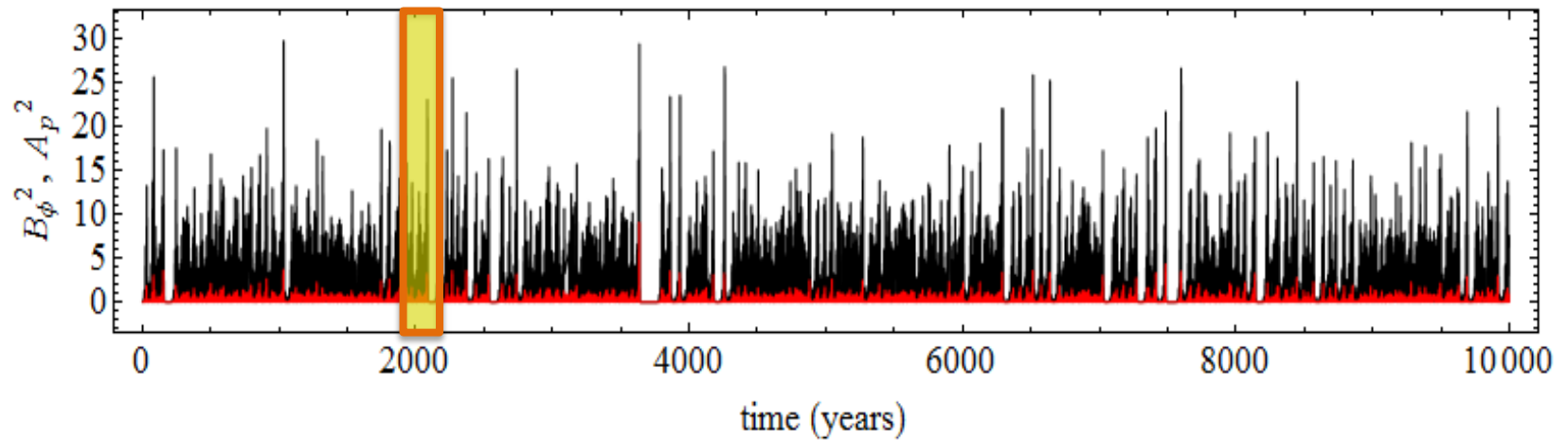


Long term modulation

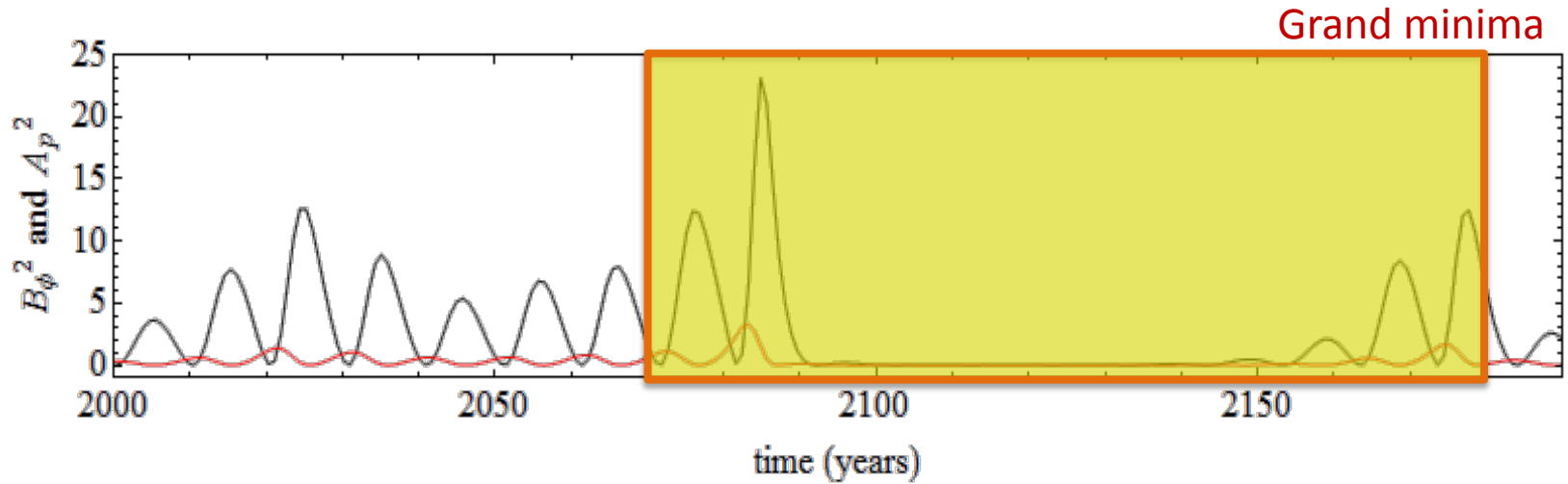


## Stochastic Fluctuations

$\tau = 1$  year,  $\mathbf{a} \in [0.01, 0.04]$ ,  $\mathbf{b} = 0.05$



# “Zooming in” into grand minima



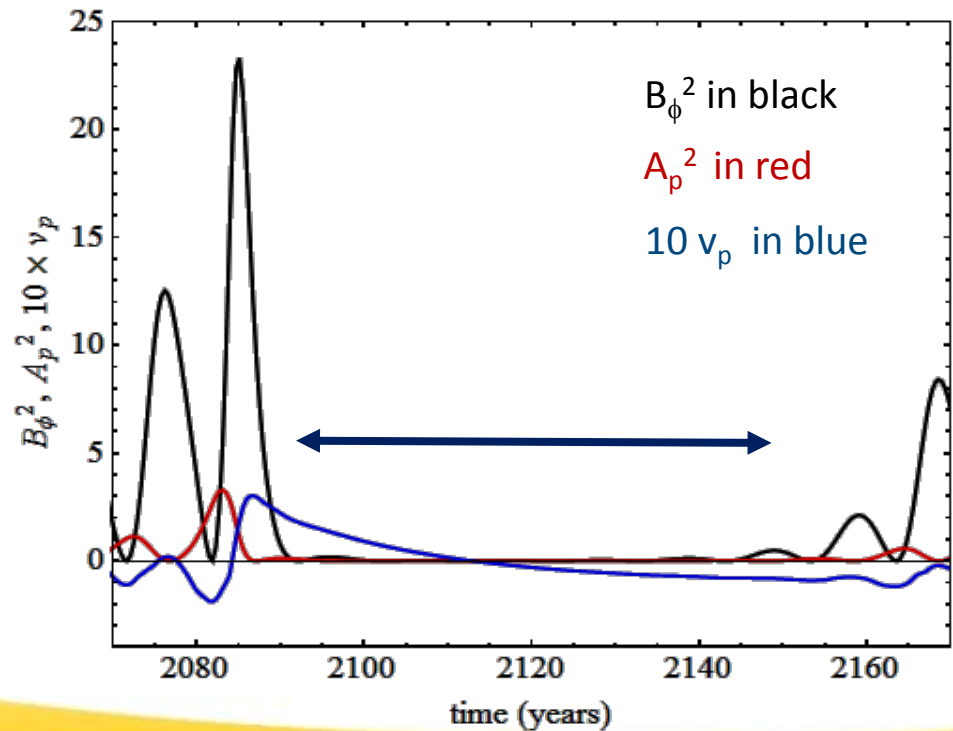
$$\frac{dv(t)}{dt} = a B_\phi A_p - b v(t)$$

Abrupt entrance into minimum

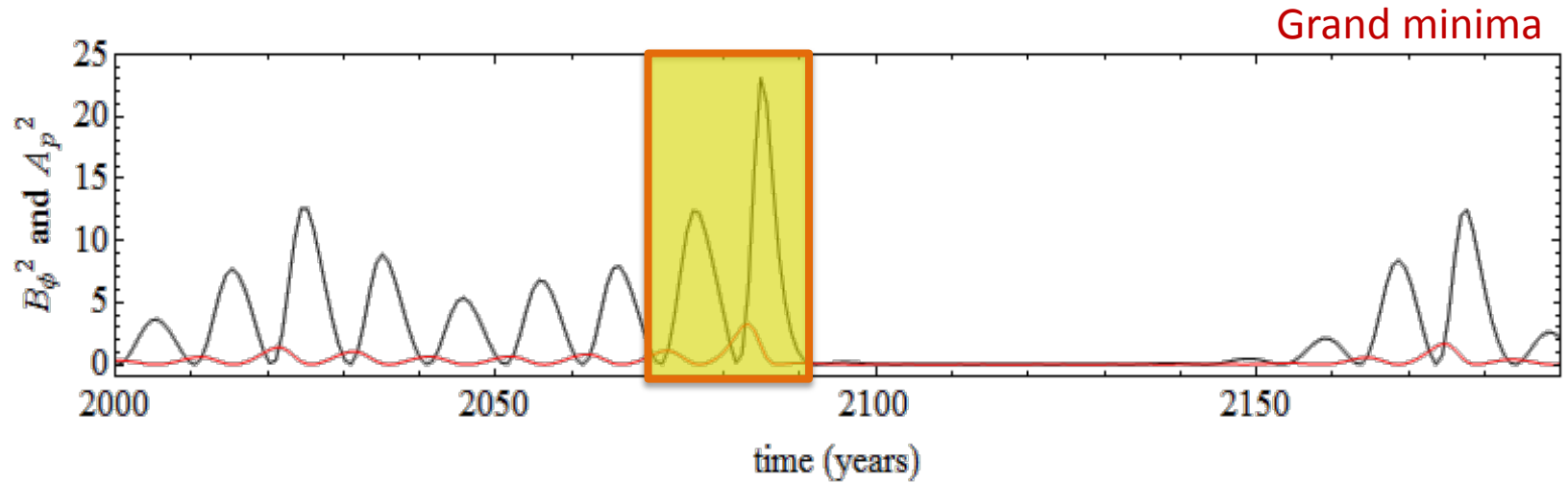
Gradual build up of mag. activity

Length of grand minima  $\propto v_p$  and  $b$ .

Higher values of  $b \Rightarrow$  shorter minima



# “Zooming in” into grand minima

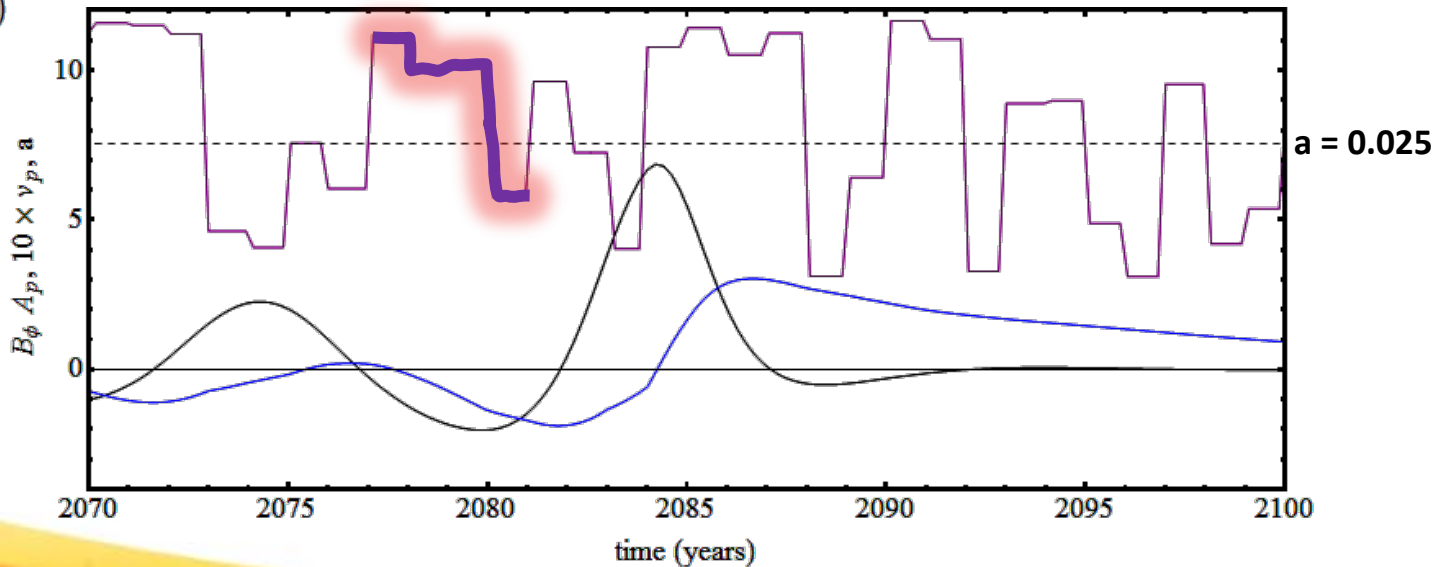


$$\frac{dv(t)}{dt} = a B_\phi A_p - b v(t)$$

$$300 \times a \in [0.01, 0.04]$$

$$10 v_p$$

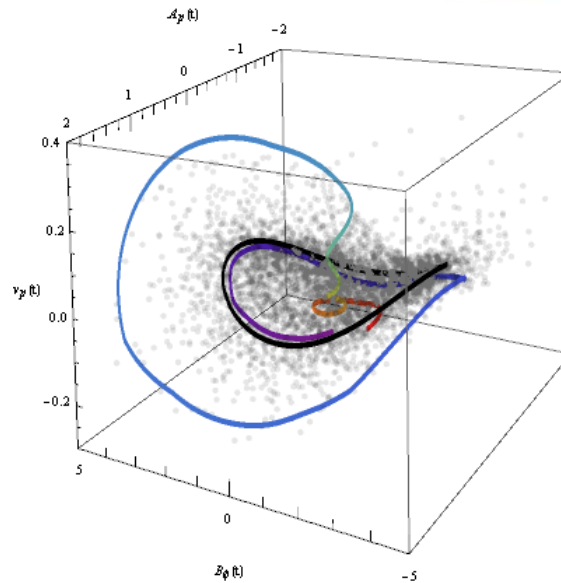
$$B(t) \quad A(t)$$





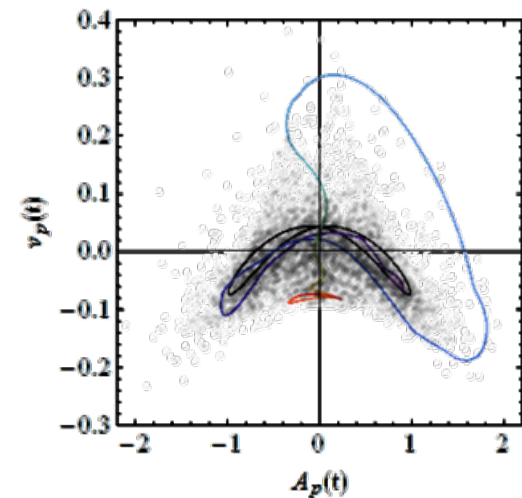
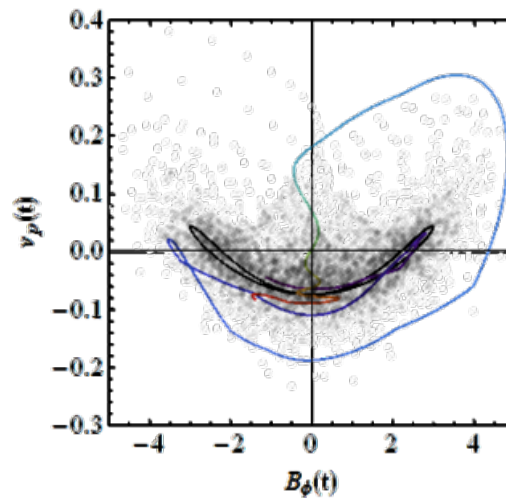
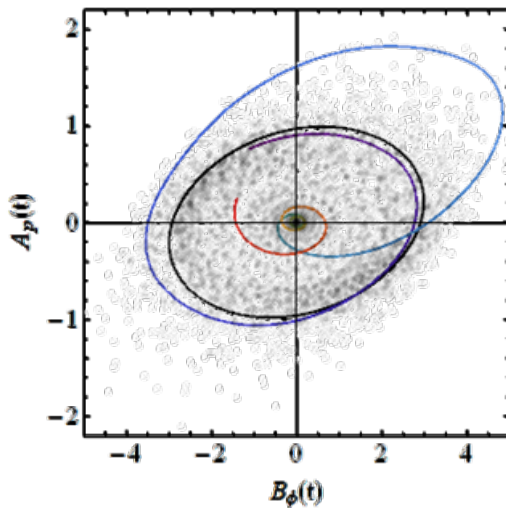
## Looking into the phase space

5000 < time < 10000  
 $a = 0.025$  in black



$a \in [0.01, 0.04]$  in gray

Minimum  
2060 < time < 2160



**Main point:** the Lorentz force feedback is important for the long term dynamics

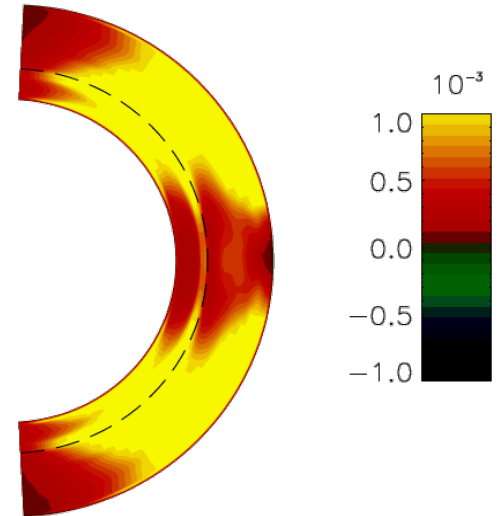
## Back to the 3D simulation

### Lorentz force

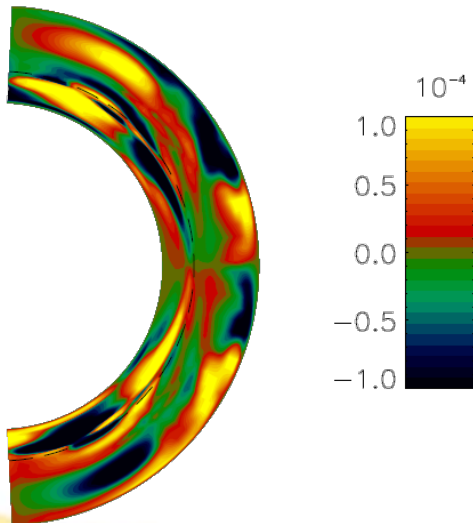
$$\mathbf{F} = \mathbf{j} \times \mathbf{B} = \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B}$$

- Compute  $\mathbf{F}$  at every grid point
- Average over all longitudes ( $\phi$ )

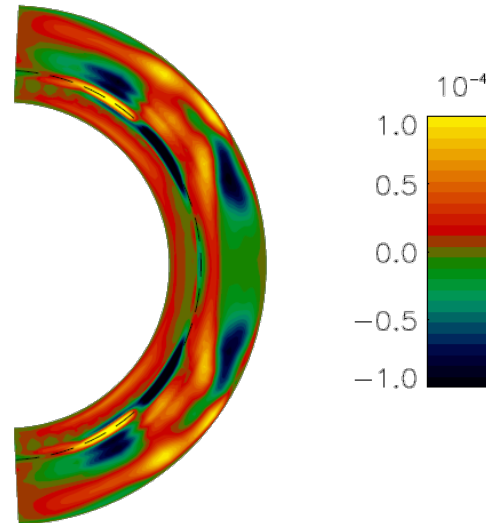
Time averaged  $\langle \mathbf{F} \rangle$



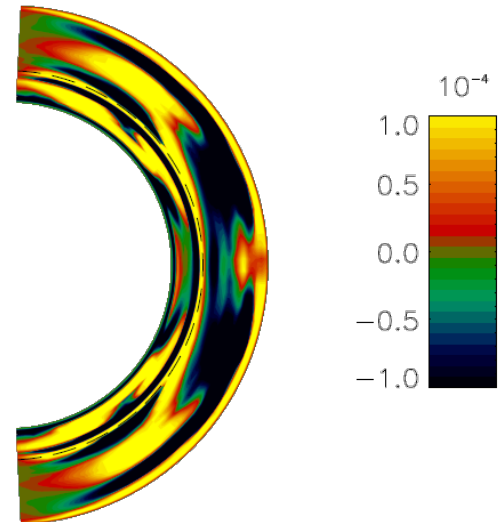
(C)  $\langle F_y \rangle$



(B)  $\langle F_x \rangle$



(D)  $\langle F_z \rangle$



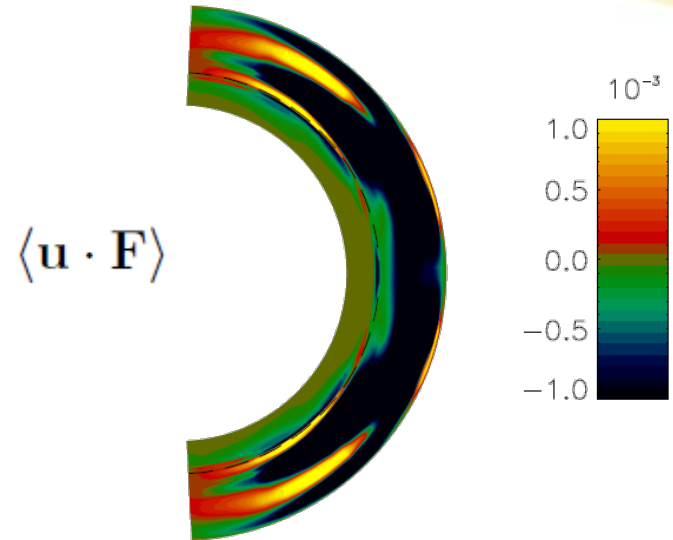
# Energy balance

## Magnetic energy evolution

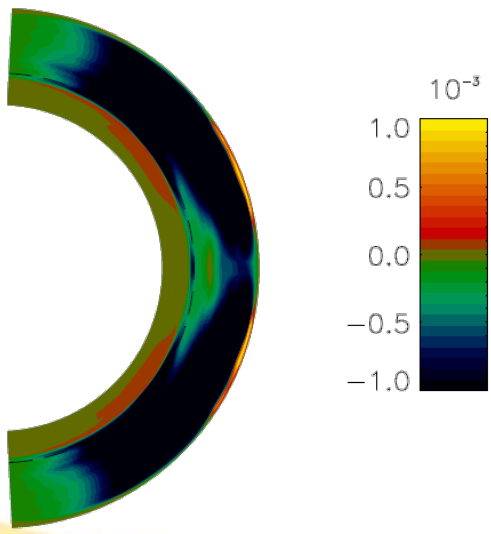
$$\frac{\partial \epsilon_B}{\partial t} = - \int_V \mathbf{u} \cdot \mathbf{F} dV - \int_V \frac{j^2}{\sigma} dV$$

Magnetic Fields gets energy from differential rotation and puts energy into the meridional flow...

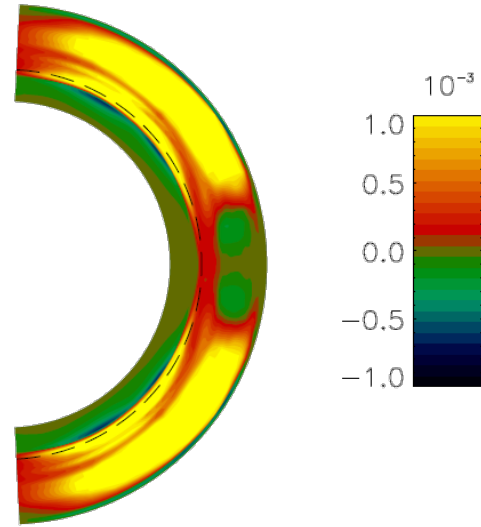
Time averaged  $\langle \mathbf{u} \cdot \mathbf{F} \rangle$



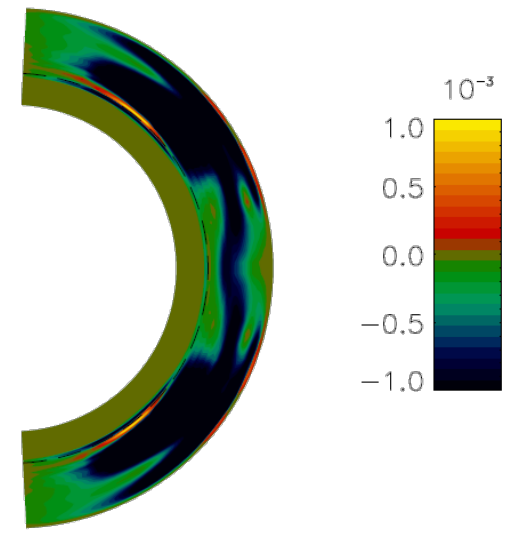
(B)  $\langle u.F_x \rangle$



(C)  $\langle v.F_y \rangle$



(D)  $\langle w.F_z \rangle$



## Mean field enough?

“Mean field” type decomposition

$$\mathbf{B} = \langle \mathbf{B} \rangle + \mathbf{B}'$$

$$\mathbf{F} = \frac{1}{\mu_0} (\nabla \times (\langle \mathbf{B} \rangle + \mathbf{B}')) \times (\langle \mathbf{B} \rangle + \mathbf{B}')$$

$$\mathbf{F} = \langle \mathbf{F} \rangle + \langle \mathbf{F}'' \rangle$$

$$\langle \mathbf{u} \cdot \mathbf{F} \rangle = \langle \mathbf{u} \cdot \mathbf{F} \rangle + \langle \mathbf{u} \cdot \mathbf{F}'' \rangle$$

$$\langle \mathbf{B} \rangle = \langle \mathbf{B} \rangle_\phi$$

$$\langle \mathbf{B}' \rangle = 0$$

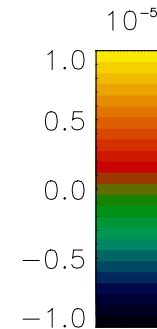
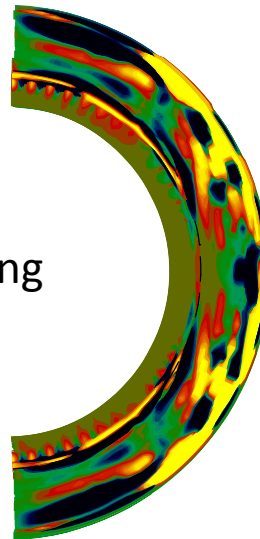
$$\langle \mathbf{B}' \mathbf{B}' \rangle \neq 0$$

### Preliminary conclusion:

For the meridional flow, the small scales have a larger impact in modelling the solar magnetic field.

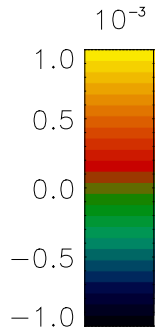
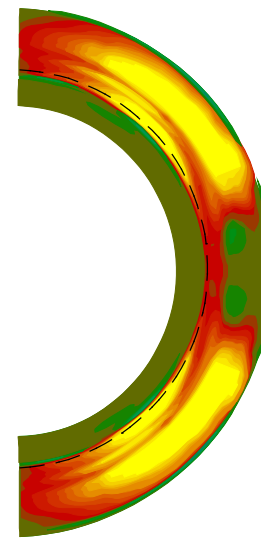
*Large scale*

$\langle C \rangle < v.FI_y >$



*Small scale*

$\langle C \rangle < v.Fs_y >$



## Conclusions (...some preliminary!!)

- ✓ Meridional flow changes with time (observed in the solar surface and in global MHD sims)
- ✓ Flux transport dynamo models working in the kinematic disregard this important info
- ✓ The Lorentz force feedback of the field into the flow generates important long term dynamic behavior...
- ✓ ...that coupled with stochastic fluctuations can even produce grand minima episodes
- ✓ Magnetic field takes energy from solar differential rotation and puts energy into the flows in the meridional plane
- ✓ The biggest contribution to this phenomena comes from the small scale field fluctuations
  
- ✓ **Main conclusion:** the results obtained through kinematic models must be taken lightly . These models should include a Lorentz force feedback term in order to be used for prediction purposes or long term studies of solar activity.

*Passos, Charbonneau, Beaudoin, Sol.Phys, 2011/12  
(in preparation)*

