Boosted black string bombs

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Outline

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2. Boosted black strings
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Motivation: BH superradiance

Superradiant instability for Kerr black holes

$$\omega < m\Omega$$

(wave analogue of Penrose process)

Black hole bomb [Press & Teukolsky (1972); Cardoso *et al.* (2004)]

- multiple scatterings off mirror around BH
- exponential production of radiation destroys mirror

Motivation: BH superradiance

Realistic "mirrors":

- black holes in AdS
- massive boson field bound states

$$\omega < \mu \sim r_+^{-1}$$

Interesting probe of BSM physics:

$$\mu \lesssim 10^{-10}\ \mathrm{eV}$$

Motivation: string axions

Strong CP problem

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\theta}{32\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} + \\ + \bar{\psi}\left(i\gamma^\mu D_\mu - m_\psi e^{i\theta'\gamma_5}\right)\psi$$

Neutron EDM constraints:

$$\bar{\theta} \lesssim 10^{-10}$$

Could rotate this term away for massless up quarks

Motivation: string axions

Peccei-Quinn solution [Peccei & Quinn (1977)]

- spontaneously broken $U(1)_{PQ}$ symmetry
- pseudo-NG boson is the axion field

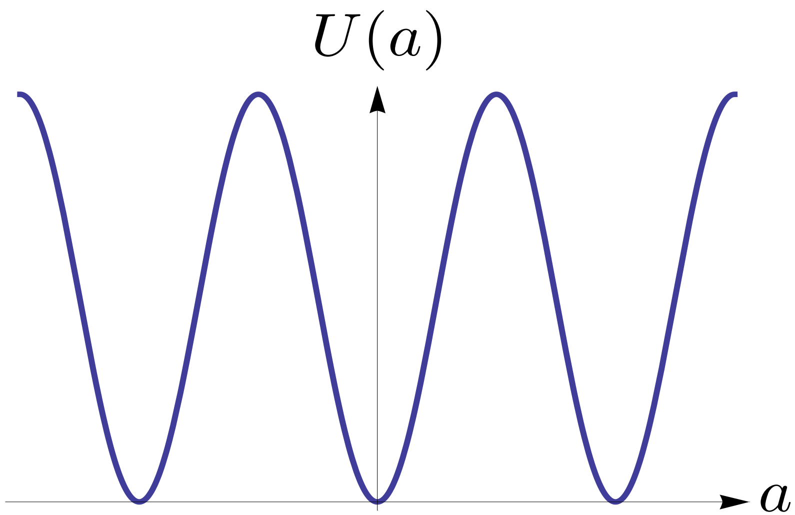
$$H = he^{ia} \,, \qquad a \to a + \theta$$

- axion potential from QCD instanton effects:

$$\mathcal{L} = \frac{1}{2}\partial_\mu a \partial^\mu a - U(a) + \frac{a}{32\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Motivation: string axions

Dynamical solution to the strong CP problem:



Axion mass is very small:

$$m_a \simeq 6 \times 10^{-10}\ \mathrm{eV} \left(\frac{10^{16}\ \mathrm{GeV}}{f_a} \right)$$

Motivation: string axions

There are many axion-like fields in string theory

$$a = \int_{C_p} B_p$$

[Arvanitaki *et al.* (2009)]

Other non-perturbative effects must be suppressed-dominant for the QCD axion

String axiverse of ultra-light fields!

Motivation: hidden photons

More exotic states?

- Superradiant bound states for bosonic fields
- Massive (hidden) photons?
  - Schwarzschild [JGR & Dolan (2011)]
  - Slowly-rotating Kerr [Pani *et al.* (2012)]
  - Extremal Kerr [Witek *et al.* (2012?)]

Motivation: black strings/branes

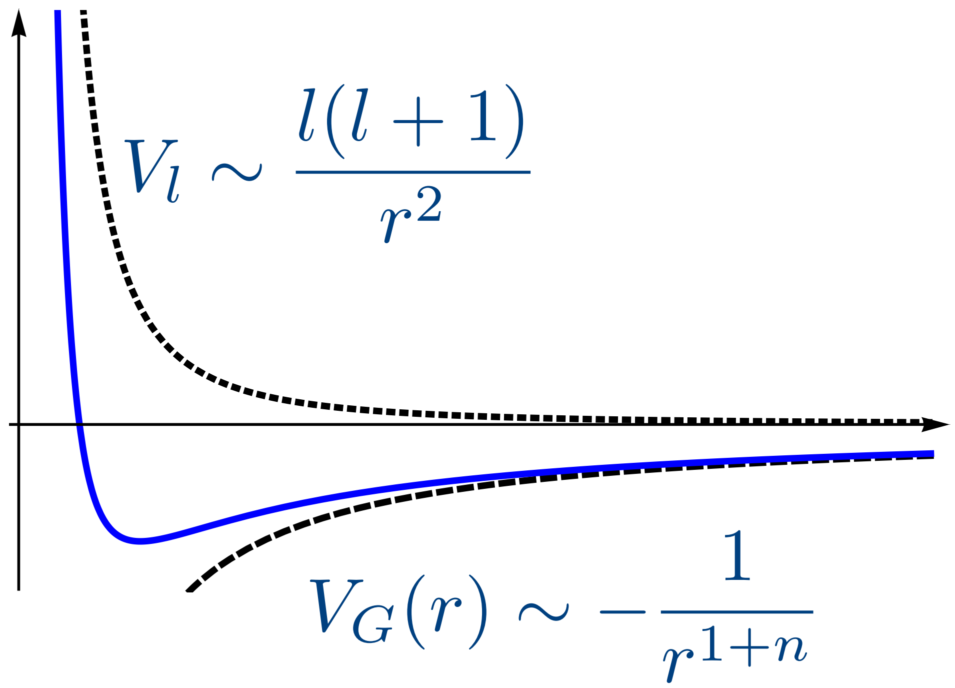
String axions arise from compactification of extra-dimensions

Black hole bombs in higher dimensions

- superradiant scattering in general
- bound states only for black strings/branes

$$\mathrm{Kerr}_4 \times \mathcal{M}_n$$

Motivation: black strings/branes



[Cardoso & Lemos; Cardoso & Yoshida (2005)]

Motivation: black strings/branes

Are black strings realistic?

- should form from collapse of matter in $\mathbb{R}_4 \times \mathcal{M}_n$

[Horowitz & Strominger (1991)]

- astrophysical BH much larger than extra-dimensions

- localized BH cannot consistently intersect braneworlds [Chamblin, Hawking & Reall (1991)]

Motivation: black strings/branes

With compact extra-dimensions, massless fields can form Kaluza-Klein bound states:

$$\mu_{KK} = \frac{n}{R}$$

But in realistic models...

$$\mu_{KK} \gtrsim \mathrm{TeV}$$

[exception: Randall-Sundrum II]

Still need ultra-light massive fields!

Motivation: black strings/branes

Can extra-dimensions modify the black hole bomb mechanism?

- simple compactification No
- non-trivial dynamics Yes!

Boosted black strings in 5D

Boosted Kerr black strings

Line element in Boyer-Lindquist coordinates:

$$
\begin{aligned}
ds^2 &= -\left(1-\frac{2Mrc_\sigma^2}{\Sigma}\right)dt^2+\frac{4Mrs_\sigma c_\sigma}{\Sigma}dtdz+\left(1+\frac{2Mrs_\sigma^2}{\Sigma}\right)dz^2 \\
&+ \frac{\Sigma}{\Delta}dr^2+\Sigma d\theta^2+\frac{(r^2+a^2)^2-\Delta a^2 s_\theta^2}{\Sigma}s_\theta^2 d\phi^2 \\
&- \frac{4Mrc_\sigma}{\Sigma}as_\theta^2 dtd\phi-\frac{4Mrs_\sigma}{\Sigma}as_\theta^2 dzd\phi
\end{aligned}
$$

$$\Delta = r^2+a^2-2Mr\ , \qquad \Sigma = r^2+a^2\cos^2\theta$$

Boosted Kerr black strings

Horizons:

$$r_{\pm} = M \pm \sqrt{M^2 - a^2}$$

Ergosurface:

$$r_e = M c_\sigma^2 + \sqrt{M^2 c_\sigma^4 - a^2 c_\theta^2}$$

Angular and linear velocity:

$$\Omega = \frac{a \cosh \sigma}{r_+^2 + a^2} \, , \qquad V_z = - \tanh \sigma$$

Boosted Kerr black strings

Mass, angular momentum and KK-charge:

$$
\begin{aligned}
M_{BS} &= ML\left(\frac{\cosh^2\sigma + 1}{2}\right) , \\
J_{BS} &= aML\cosh\sigma , \\
P_{BS} &= \frac{ML}{4}\sinh(2\sigma) .
\end{aligned}
$$

Note: Lorentz symmetry broken for compact extra-dimension [Lee & Kim (2008)]

Boosted Kerr black strings

Modified superradiant condition [Dias (2006)]

$$\omega < \frac{m\Omega}{c_\sigma^2} - k t_\sigma$$

What about massive field bound states?

Massive scalar bound states

Klein-Gordon equation:

$$(\nabla_\mu \nabla^\mu - \mu^2)\Phi = 0$$

Separation of variables:

$$\Phi(t, r, \theta, \phi, z) = \sum e^{i(-\omega t + m\phi - kz)} S_{lm}(\theta) R_{nlm}(r)$$

Massive scalar bound states

Angular equation:

$$\frac{1}{\sin\theta}\partial_\theta\left(\sin\theta\,\partial_\theta S\right)+\left[a^2(\omega^2-\mu^2-k^2)\cos^2\theta-\frac{m^2}{\sin^2\theta}+\lambda\right]S=0$$

Radial equation:

$$\begin{aligned}
&\Delta\partial_r\left(\Delta\partial_r R\right)-\Delta\left[(\mu^2+k^2)r^2+a^2\omega^2-2\omega m a c_\sigma+\lambda\right]R+\\
+\quad&\left[\left(\omega(r^2+a^2)-mac_\sigma\right)^2+2Mr(r^2+a^2)c_\sigma^2(\omega-kt_\sigma)^2\right.\\
-\quad&\left.2Mr(r^2+a^2)\omega^2-m^2a^2s_\sigma^2+4kmaMrs_\sigma\right]R=0
\end{aligned}$$

Massive scalar bound states

Angular eigenvalue:

$$\lambda = l(l+1) + \sum_{j=1}^{\infty} c_{jlm} (aq)^{2j}$$

where

$$q = \sqrt{\mu^2 + k^2 - \omega^2}$$

Bulk mass KK mass

Massive scalar bound states

Dimensionless variables:

$$x = \frac{r - r_+}{r_+} \qquad\qquad \tau = \frac{r_+ - r_-}{r_+}$$

in terms of which we have:

$$\left[x^2(x+\tau)^2\partial_x^2 + x(x+\tau)(2x+\tau)\partial_x + V(x)\right]R = 0$$

Analytical results

Matching procedure [Starobinsky (1973)]

- Near region: $\bar{\omega} x \ll l$
- Far region: $x \gg 1$
- Matching region: $1 \ll x \ll l/\bar{\omega}$

Note: valid for small mass only!

Analytical results

Near region:

$$V(x) \simeq \bar{\varpi}^2 - \lambda x(x+\tau)$$

$$\bar{\varpi} = (2-\tau)c_\sigma \left(\bar{\omega} - \frac{m\bar{\Omega}}{c_\sigma^2} - \bar{k}t_\sigma \right)$$

Solution with “ingoing” boundary conditions:

$$R_{near}(x) = A \left(\frac{x}{x+\tau} \right)^{-i\bar{\varpi}/\tau} {}_2F_1 \left(l+1, -l, 1-2i\bar{\varpi}, -\frac{x}{\tau} \right)$$

Analytical results

Far region:

$$x^2\partial_x^2 R + 2x\partial_x R + (-\bar{q}^2 x^2 + 2\bar{q}\nu x - \lambda)R = 0$$

$$\nu = \frac{(2-\tau)}{2}\frac{(\bar{\omega}c_\sigma - \bar{k}s_\sigma)^2 - \bar{q}^2}{\bar{q}}$$

Bound state solution:

$$R_{far}(x) = Bx^l e^{-\bar{q}x} U(l+1-\nu, 2l+2, 2\bar{q}x)$$

Analytical results

Matching region:

$$R_{near}(x) \simeq A\Gamma(1-2i\bar{\varpi})\left[\frac{\Gamma(2l+1)}{\Gamma(l+1)\Gamma(l+1-2i\bar{\varpi})}\left(\frac{x}{\tau}\right)^{l}+\frac{\Gamma(-2l-1)}{\Gamma(-l)\Gamma(-l-2i\bar{\varpi})}\left(\frac{x}{\tau}\right)^{-l-1}\right]$$

$$R_{far}(x) \simeq B\frac{\pi}{\sin((2l+2)\pi)}\left[\frac{x^{l}}{\Gamma(-l-\nu)\Gamma(2l+2)}-\frac{(2\bar{q})^{-(2l+1)}x^{-l-1}}{\Gamma(l+1-\nu)\Gamma(-2l)}\right]$$

Matching condition:

$$\frac{\Gamma(-l-\nu)\Gamma(2l+2)}{\Gamma(l+1-\nu)\Gamma(-2l)}=-(2\bar{q}\tau)^{2l+1}\frac{\Gamma(-2l-1)}{\Gamma(-l)}\frac{\Gamma(l+1)}{\Gamma(2l+1)}\frac{\Gamma(l+1-2i\bar{\varpi})}{\Gamma(-l-2i\bar{\varpi})}$$

Analytical results

Lowest order:

$$\nu_0 = l + 1 + n$$

Hydrogen-like spectrum for zero-modes:

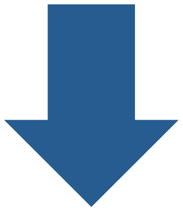
$$\omega \simeq \mu \left(1 - \frac{(\mu M)^2 c_\sigma^4}{2(l + 1 + n)^2} \right)$$

Note: valid for small boost only!

Analytical results

Imaginary part (NLO):

$$lim_{z\to -n}\frac{\psi(z)}{\Gamma(z)} = (-1)^{n+1}n!\ , \qquad \frac{\Gamma(-2l-1)}{\Gamma(-l)} = \frac{(-1)^{l+1}}{2}\frac{l!}{(2l+1)!}\ , \qquad \frac{\Gamma(l+1+x)}{\Gamma(-l-x)} = (-1)^l x\prod_{j=1}^{l}(j^2-x^2)$$



$$\mathrm{Im}(\omega M) = -\frac{1}{2}C_{ln}(\varpi M)(\mu M)^{4l+5}c_\sigma^{4l+6}\left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{2l+1}$$

$$C_{ln} = \frac{4^{2l+2}}{(l+1+n)^{2l+4}}\frac{(2l+1+n)!}{n!}\left(\frac{l!}{(2l+1)!(2l)!}\right)^2\prod_{j=1}^{l}(j^2+16(\varpi M)^2)$$

Analytical results

Fastest growing mode $(l = m = 1\ , n = 0)$

$$\mathrm{Im}(\omega M) \simeq \frac{1}{48}\left(\frac{a}{M}c_\sigma\right)(\mu M c_\sigma)^9$$

in the slow rotation limit.

Boost enhancement!

Note: factor 2 difference with Detweiller (1980) and Furuhashi & Nambu (2004) but agreement with Pani *et al.* (2012).

Numerical results

Forward integration method:

1. Near-horizon expansion with ingoing b.c.

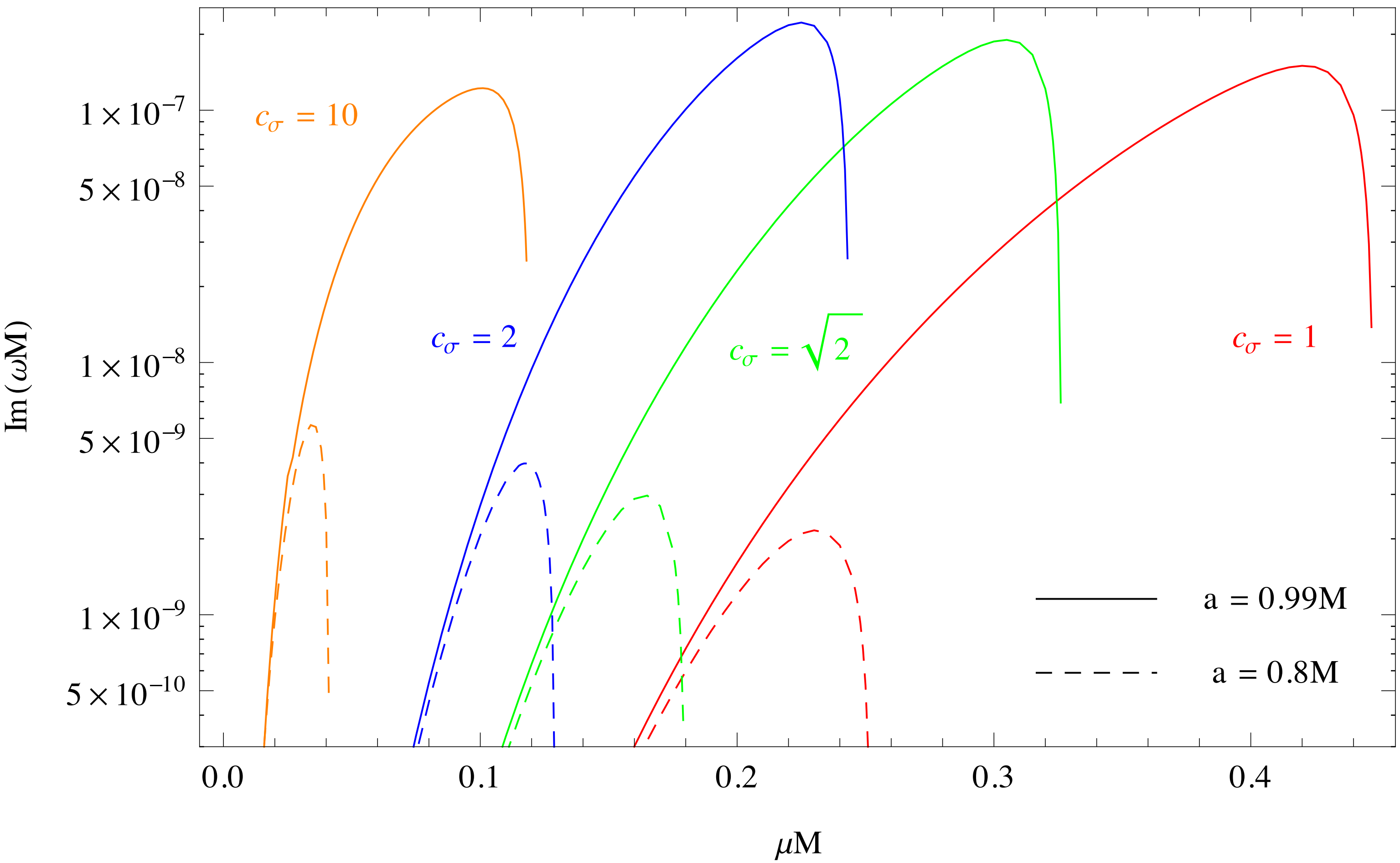
$$R(x) = x^{-i\bar{\varpi}/\tau} \sum_{n=0}^{\infty} a_n x^n$$

2. Numerically integrate up to large distance

3. Minimize asymptotic wave-function to obtain bound states

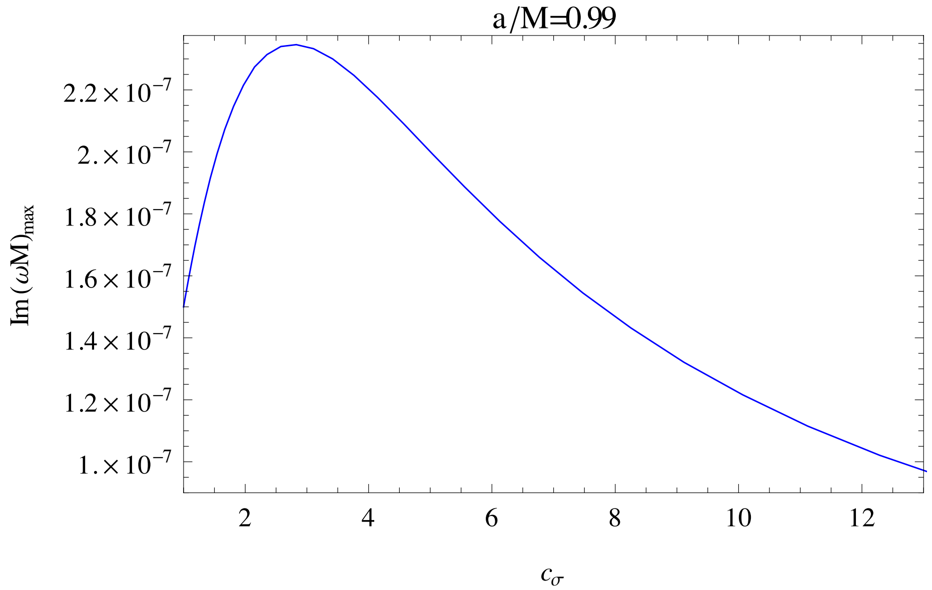
[JGR & Dolan (2011)]

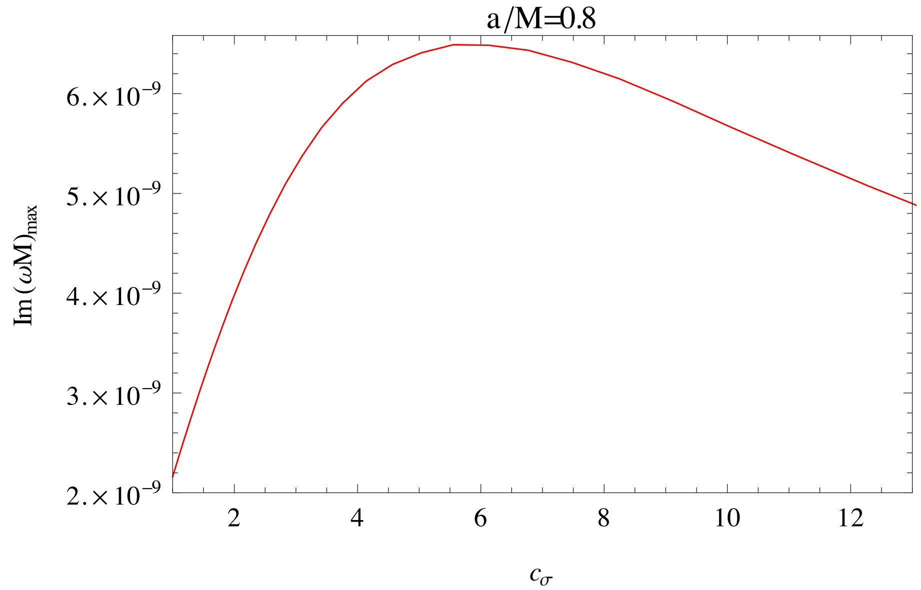
Numerical results



Numerical results

Maximum growth rate

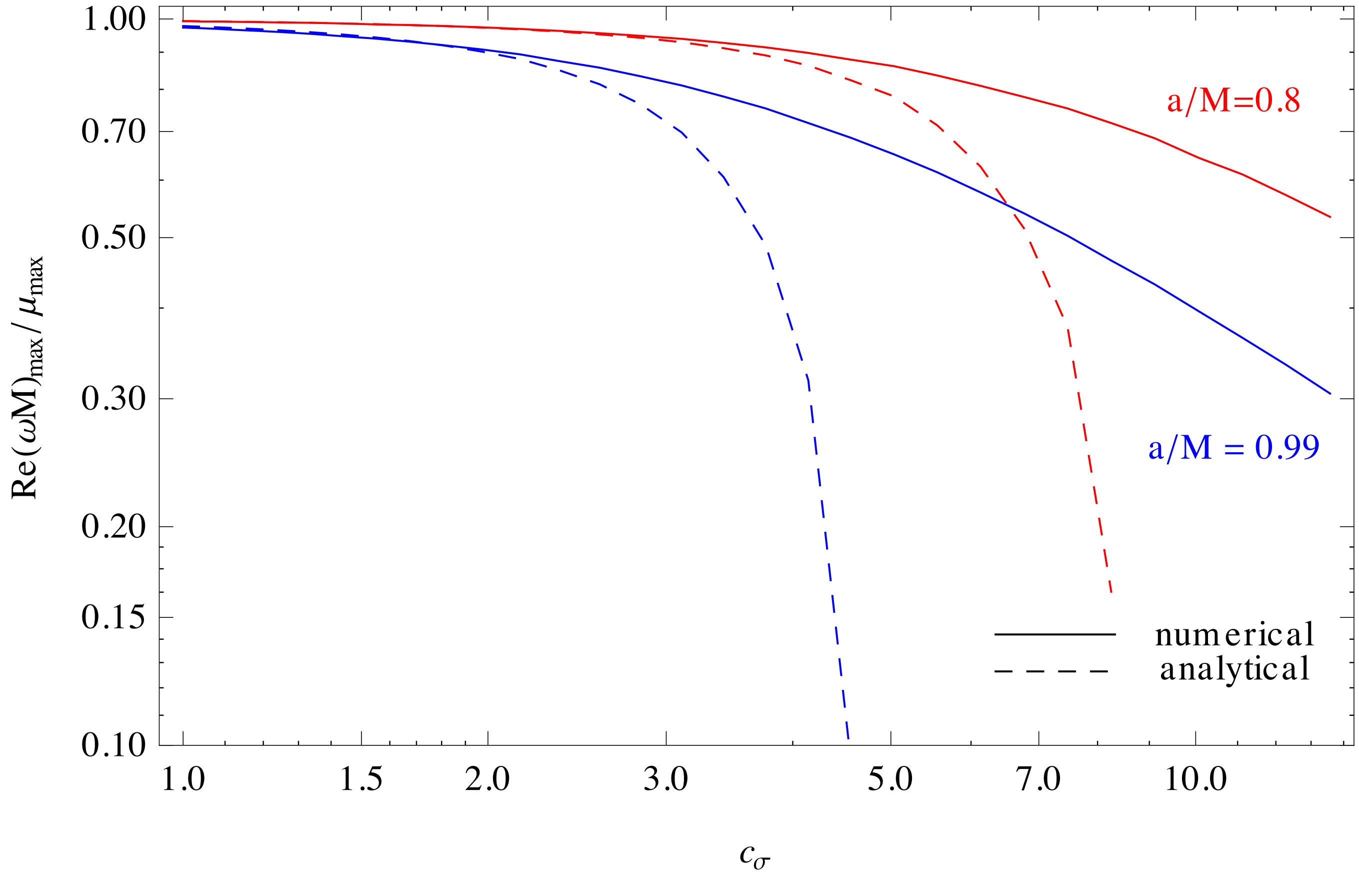




$$\mathrm{Im}(\omega M)_{\mathrm{max}} \simeq \frac{\alpha c_\sigma}{1+\beta c_\sigma^2}$$

Numerical results

Deviation from Hydrogen-like spectrum



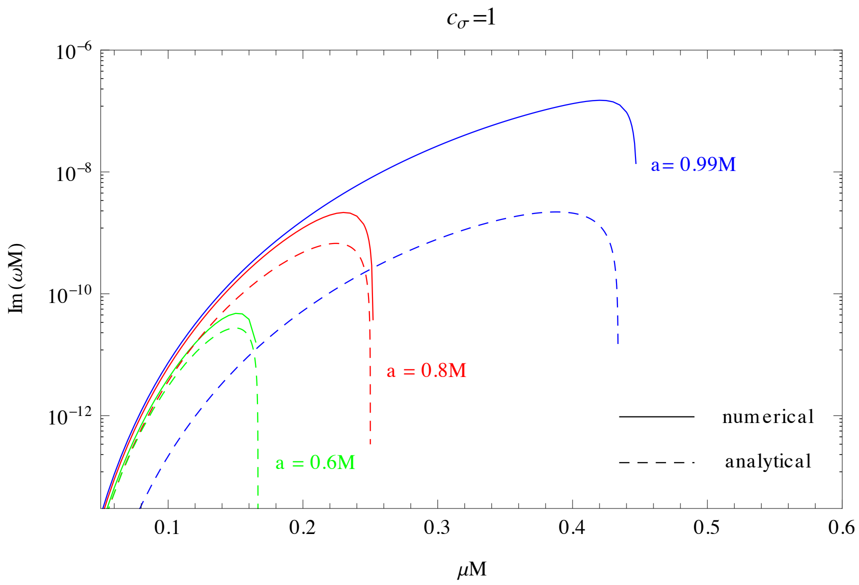
Numerical results

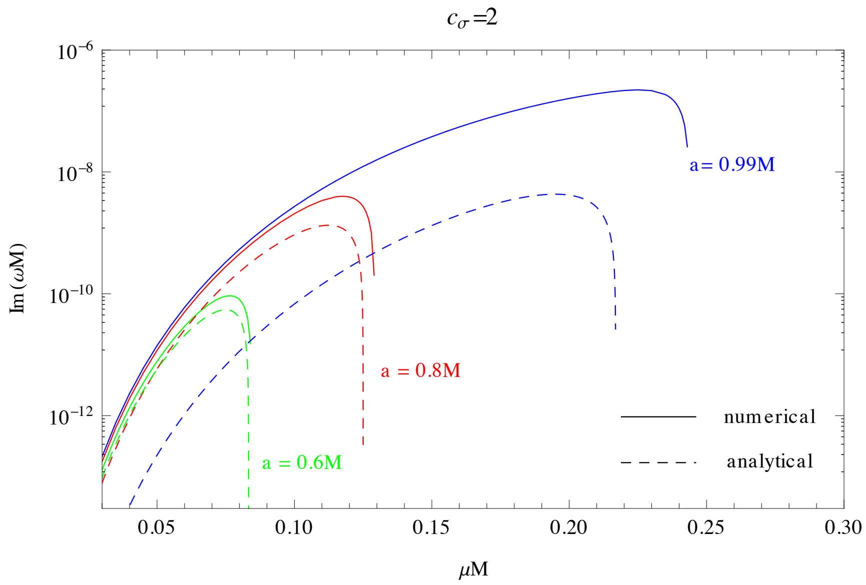
Conclusions:

- States not as tightly bound as matching suggests
- Superradiant growth rate:
  - increases due to boundedness
  - decreases for smaller masses

Numerical results

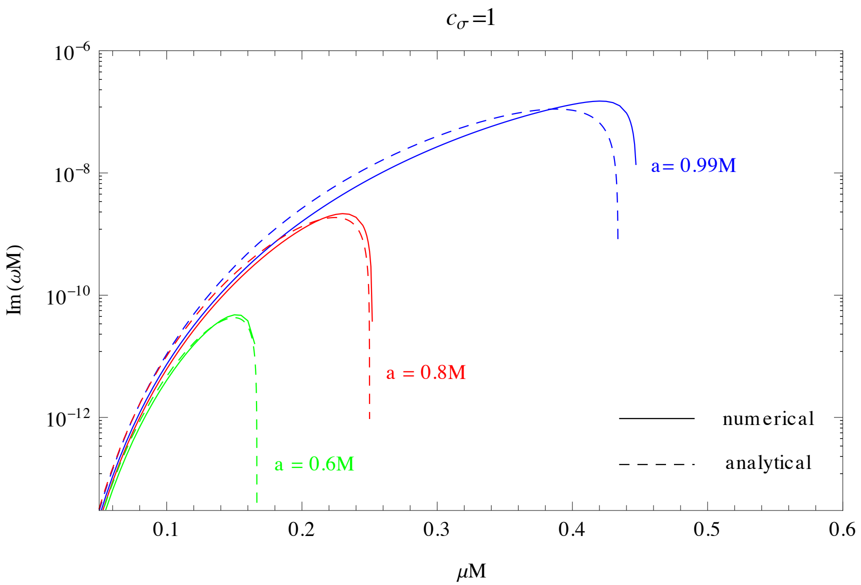
Comparison with matching

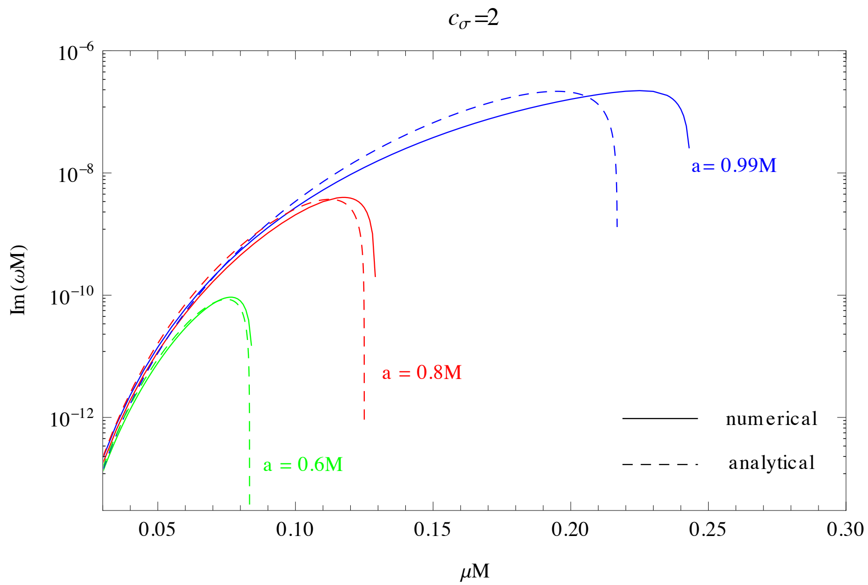




Numerical results

Comparison with matching





Remove *l*-dependence in $\left(\frac{r_+ - r_-}{r_+ + r_-}\right)^{2l+1}$

Dimensional reduction

KK-decomposition: [Kunz *et al.* (2006)]

$$ds^2 = e^{2\alpha\varphi} ds_4^2 + e^{-4\alpha\varphi}(dz + A_\mu dx^\mu)^2$$

$$\begin{aligned}
ds_4^2 &= e^{-2\alpha\varphi}\left[-dt^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + (r^2+a^2)\sin^2\theta d\phi^2 + e^{4\alpha\varphi}\frac{2Mr}{\rho^2}\left(c_\sigma dt - a\sin^2\theta d\phi\right)^2\right] \ , \\
A_\mu dx^\mu &= \frac{2Mr}{\rho^2} s_\sigma e^{4\alpha\varphi}\left(c_\sigma dt - a\sin^2\theta d\phi\right) \ , \\
e^{-4\alpha\varphi} &= 1 + \frac{2Mr}{\rho^2} s_\sigma^2 \ .
\end{aligned}$$

Dimensional reduction

Comparison with Kerr-Newman:

$$
\begin{aligned}
Q_{BS} &= P_{BS} \\
\mathcal{M}_{BS} &= a s_\sigma \frac{ML}{2} \ , \\
\Phi_{BS} &= \frac{s_\sigma^2}{8\alpha} ML
\end{aligned}
$$

Dimensional reduction

Superradiance condition: [Furuhashi & Nambu (2004)]

$$\omega < m\Omega + q\tilde{Q}$$

Which leads to the identification:

$$q = 4k/c_\sigma^2$$

Due to the additional dilatonic charge, the superradiant condition is modified for zero-modes!

Observational prospects

Where does the KK-charge (boost) come from?

- BH are expected to be electrically neutral
- KK-momentum not necessarily conserved:
  - KK-momentum quantization
  - Topological properties (orbifolds, orientifolds, branes...)

Observational prospects

Universal Extra-Dimensions (UED)

• all SM fields in bulk with S^1/Z_2 orbifold

• KK-parity conservation

• natural dark matter candidate (LKP: photon, neutrino, graviton)

Could a black string absorb KK-charge from dark matter?

Observational prospects

- Dark matter has particles of opposite KK-charges
- KK-number violating interactions
  - Localized at orbifold-fixed points
  - SM fields in “thick brane” in higher-dimensions

⬇

KK-asymmetry in the early universe could yield boosted black strings

Observational prospects

Size of extra-dimension is unobservable...

$$\mu M \quad \to \quad \frac{\mu G_5 M}{\hbar c} = \frac{2}{c_\sigma^2 + 1} \frac{G_4 M_{BS} \mu}{\hbar c}$$

$$\frac{a}{M} \quad \to \quad \frac{ac}{G_5 M} = \frac{(c_\sigma^2 + 1)^2}{4 c_\sigma} \frac{J_{BS} c}{G_4 M_{BS}^2}$$

... but KK-momentum may be!

Observational prospects

Upper bound on spin-mass ratio:

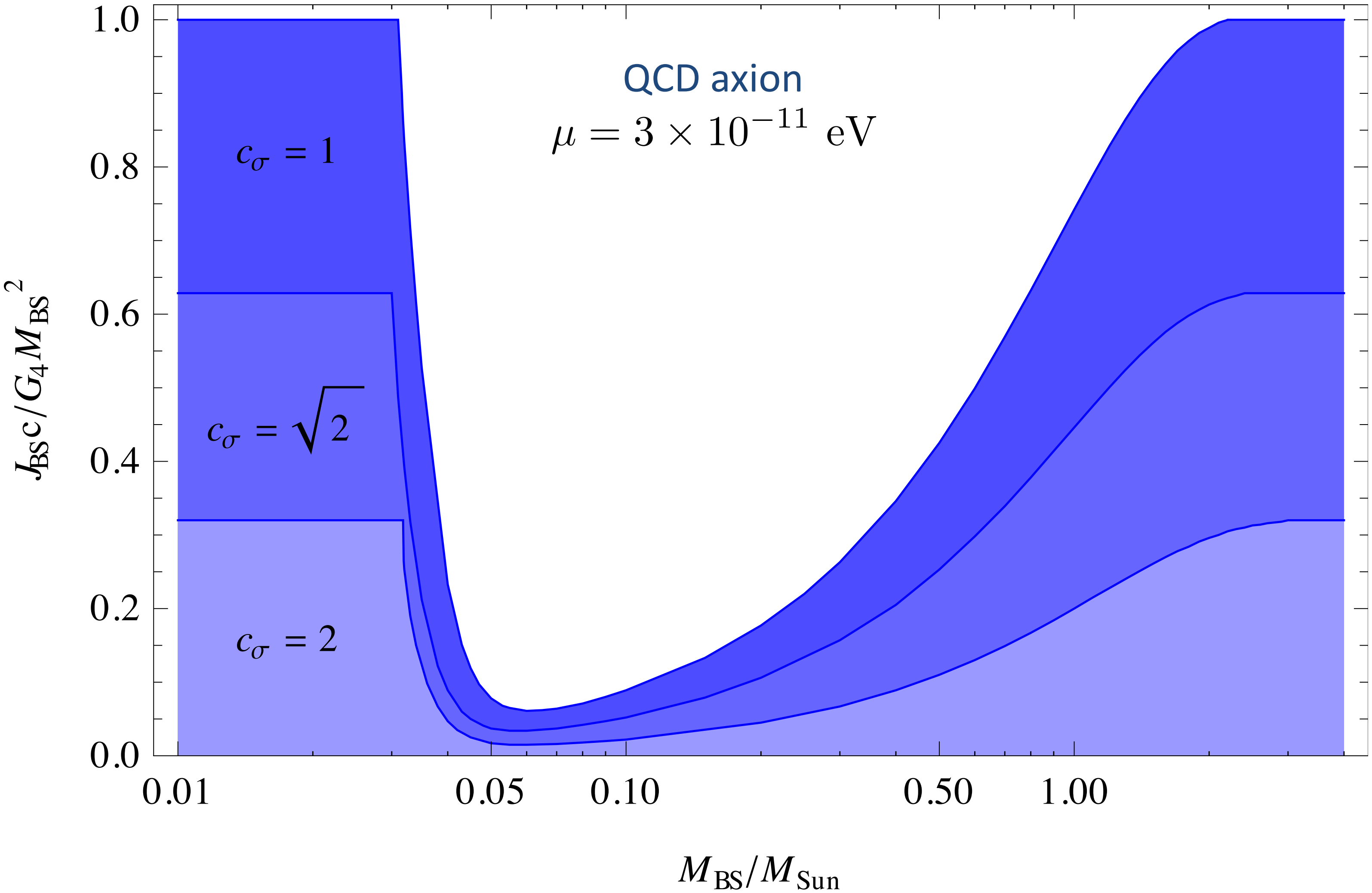
$$\frac{J_{BS}c}{G_4 M_{BS}^2} \leq \frac{4c_\sigma}{(c_\sigma^2+1)^2}$$

Gaps in Regge plot: [Arvanitaki *et al.* (2009)]

- Axions spin-down black string (> 100 e-folds)
- Accretion spins-up black string:

$$\tau_E = \sigma_T / 4\pi G m_P \sim 4 \times 10^8 \text{ yrs}$$

Observational prospects



Observational prospects

Experimental bounds: [McClintock *et al.* (2009)]

- GRS 1915+15

$$Jc/G_4M^2 = 0.98 - 1 \qquad \Rightarrow \qquad c_\sigma < 1.01$$

- LMC X-3

$$Jc/G_4M^2 < 0.26 \qquad \Rightarrow \qquad c_\sigma \gtrsim 2$$

Young BH aligned along constant boost curves?

Summary

- Superradiant bound states possible for black strings/branes

- Non-trivial dynamics changes superradiant spectrum

- Boost enhances growth rate up to 56% (extremal)

- Probe of non-trivial compactifications

- Independence of extra-dimensional size (complementary to LHC)

Still a lot to do!