The dynamics of Modified Gravity

Vincenzo Vitagliano

15th Dec 2011
SISSA, Trieste & (asap...) CENTRA, Lisboa

Introduction

Why Modified Gravities? Why now?
GR notably succeed in explaining observations.
Intrinsic limits of GR: existence of solutions with curvature singularities, closed time-like loops...
$\Longrightarrow$ Hints that the theory had to be replaced in some very strong field limit.

Need for a modified theory whose relevance was going to be limited to very strong gravity phenomena.

But in the last two decades...
Cosmological constant? Current available theories of particles?
Why not a viable alternative theory of gravity?

Introduction

A very broad variety of models!
The form of the gravitational action has been among the most questioned.

A quantum theory of gravity requires a generalization of the Einstein-Hilbert action.
Phenomenological interest: one might replace the scalar curvature by some function of curvature invariants which could then be expanded in a power series...

$$f(R, R_{\mu\nu}R^{\mu\nu}, R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}, \ldots)!!!$$

Introduction

In GR spacetime geometry is fully described by the metric:
it does not only define distances, which is its primary role, but also defines parallel transport.

However, this does not have to be a *condicio sine qua non*.
The metric and the connection can be independent quantities.

Metric variation $\Rightarrow$ Palatini approach
WARNING!
The matter action still does not depend on the connection!!!
MAG: the independent connection is allowed to enter in the matter sector of the Lagrangian
(e.g. Einstein-Cartan theory).

Ouverture: variational approaches

Both standard metric and Palatini variations of EH action lead to equivalent systems of field equations.

For more general actions, this is not true anymore!
Widely studied example: $f(R)$ theories of gravity

$$S = \frac{1}{l_p^2}\int dx^4 \sqrt{-g} f(\mathcal{R}) + S_M(\psi, g_{\mu\nu})$$

- metric field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)\, f'(R) = \kappa T_{\mu\nu}$$

- Palatini field equations

$$f'(\mathcal{R})\mathcal{R}_{(\mu\nu)} - \frac{1}{2}f(\mathcal{R})g_{\mu\nu} = \kappa T_{\mu\nu}$$

$$\nabla_\lambda\left(\sqrt{-g}f'(\mathcal{R})g^{\mu\nu}\right) = 0$$

Ouverture: variational approaches

Both are metric theories in the sense that
i) gravity is associated to a symmetric tensor (the metric),
ii) the response of matter and fields to gravity is described by $\nabla_\mu T^{\mu\nu} = 0$.
In any case, the two theories are not equivalent.

The *a priori* independent connection in Palatini $f(R)$ gravity does not carry any dynamics: it is an auxiliary field that can algebraically eliminated.

Palatini $f(R)$ dynamically equivalent to Brans-Dicke theory with $\omega_0 = -3/2$ (a particular class in which the scalar field doesn't add any new dynamics).

Murder, he wrote...

Eliminating the (symmetric and not) connection in Palatini

$$\Gamma^{\lambda}{}_{\mu\nu} = \left\{{}^{\lambda}{}_{\mu\nu}\right\} + \frac{1}{2f'}\left[2\partial_{(\mu} f' \delta^{\lambda}_{\nu)} - g^{\lambda\sigma} g_{\mu\nu} \partial_{\sigma} f'\right]$$

$$f'(\mathcal{R})\mathcal{R} - 2f(\mathcal{R}) = \kappa T$$

The algebraic equation can be generically solved to give $\mathcal{R}(T)$.
Some exceptions: $f \propto \mathcal{R}^2$, no root of the equation...
Cook everything into the expression of Γ...

Main tools

The covariant derivative of the connection $\Gamma^{\rho}{}_{\mu\nu}$ acting on a tensor is defined as

$$\nabla_{\mu} A^{\nu}{}_{\sigma} = \partial_{\mu} A^{\nu}{}_{\sigma} + \Gamma^{\nu}{}_{\alpha\mu} A^{\alpha}{}_{\sigma} - \Gamma^{\alpha}{}_{\sigma\mu} A^{\nu}{}_{\alpha}$$

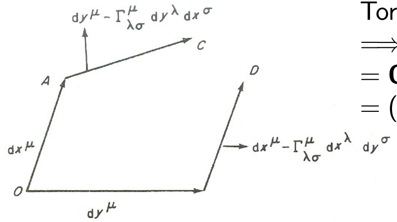
The antisymmetric part of the connection is commonly referred to as the Cartan torsion tensor

$$S_{\mu\nu}{}^{\lambda} \equiv \Gamma^{\lambda}{}_{[\mu\nu]}$$

The failure of the connection to covariantly conserve the metric is measured by the non-metricity tensor

$$Q_{\lambda\mu\nu} \equiv -\nabla_{\lambda} g_{\mu\nu}$$

Geometrical interpretation



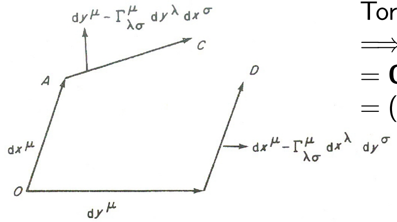
Torsion $\Rightarrow \quad \Gamma^\lambda{}_{\mu\nu} \neq \Gamma^\lambda{}_{\nu\mu}$

$\Longrightarrow \mathbf{OC} - \mathbf{OD} =$

$= \mathbf{OA} + \mathbf{AC} - \mathbf{OB} - \mathbf{BD} =$

$= (\Gamma^\mu{}_{\lambda\sigma} - \Gamma^\mu{}_{\sigma\lambda})\, dx^\lambda dy^\sigma \neq 0$

Geometrical interpretation



Torsion $\Rightarrow \quad \Gamma^{\lambda}{}_{\mu\nu} \neq \Gamma^{\lambda}{}_{\nu\mu}$

$$\Longrightarrow \mathbf{OC} - \mathbf{OD} =$$
$$= \mathbf{OA} + \mathbf{AC} - \mathbf{OB} - \mathbf{BD} =$$
$$= \left(\Gamma^{\mu}{}_{\lambda\sigma} - \Gamma^{\mu}{}_{\sigma\lambda}\right) dx^{\lambda} dy^{\sigma} \neq 0$$

$Q_{\lambda\mu\nu} \neq 0 \quad \Longrightarrow$ lengths (inner products) not preserved

$$D\left(g_{\mu\nu} u^{\mu} w^{\nu}\right) = \left(D g_{\mu\nu}\right) u^{\mu} w^{\nu} = u^{\mu} w^{\nu} \nabla_{\chi} g_{\mu\nu} d\xi^{\chi} =$$
$$= -u^{\mu} w^{\nu} Q_{\chi\mu\nu} d\xi^{\chi}$$

The revenge of Palatini theories

Actions which contains higher order curvature invariants

$$S = \frac{1}{l_p^2} \int dx^4 \sqrt{-g} f(\mathcal{R}, \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}) + S_M(\psi, g_{\mu\nu})$$

The Ricci tensor is given in term of the connection as

$$\mathcal{R}_{\mu\nu} = \partial_\lambda \Gamma^\lambda{}_{\mu\nu} - \partial_\nu \Gamma^\lambda{}_{\mu\lambda} + \Gamma^\lambda{}_{\sigma\lambda}\Gamma^\sigma{}_{\mu\nu} - \Gamma^\lambda{}_{\sigma\nu}\Gamma^\sigma{}_{\mu\lambda}$$

and for simplicity the connection is assumed to be symmetric

$$\Gamma^\rho{}_{\alpha\beta} = \left\{{}^{\rho}{}_{\alpha\beta}\right\} + \frac{1}{2} g^{\rho\lambda} \left[Q_{\alpha\beta\lambda} + Q_{\beta\alpha\lambda} - Q_{\lambda\alpha\beta}\right]$$

For a symmetric connection, it is still possible that

$$\mathcal{R}_{[\alpha\beta]} = -\partial_{[\beta}\Gamma^\lambda{}_{\alpha]\lambda} = -2\nabla_{[\beta} Q_{\alpha]}$$

The revenge of Palatini theories

Easiest case: $S = \frac{1}{l_p^2} \int dx^4 \sqrt{-g} \left[\mathcal{R} + l_p^2 \mathcal{R}_{\mu\nu} (a \mathcal{R}^{\mu\nu} + b \mathcal{R}^{\nu\mu}) \right]$

The revenge of Palatini theories

Easiest case: $S = \frac{1}{l_p^2}\int dx^4\sqrt{-g}\,\left[\mathcal{R} + l_p^2\mathcal{R}_{\mu\nu}(a\mathcal{R}^{\mu\nu} + b\mathcal{R}^{\nu\mu})\right]$

$\Rightarrow S = \frac{1}{l_p^2}\int dx^4\sqrt{-g}\left[\mathcal{R} + c_1\, l_p^2\,\mathcal{R}_{(\mu\nu)}\mathcal{R}^{(\mu\nu)} + c_2\, l_p^2\,\mathcal{R}_{[\mu\nu]}\mathcal{R}^{[\mu\nu]}\right]$

The revenge of Palatini theories

Easiest case: $S = \frac{1}{l_p^2}\int dx^4\sqrt{-g}\left[\mathcal{R} + l_p^2\mathcal{R}_{\mu\nu}(a\mathcal{R}^{\mu\nu} + b\mathcal{R}^{\nu\mu})\right]$

$\Rightarrow S = \frac{1}{l_p^2}\int dx^4\sqrt{-g}\left[\mathcal{R} + c_1\, l_p^2\,\mathcal{R}_{(\mu\nu)}\mathcal{R}^{(\mu\nu)} + c_2\, l_p^2\,\mathcal{R}_{[\mu\nu]}\mathcal{R}^{[\mu\nu]}\right]$

Field equations

$$
\begin{aligned}
i)\ \ \mathcal{R}_{(\mu\nu)} - \frac{1}{2}\mathcal{R}g_{\mu\nu} + 2c_1\, l_p^2\,\mathcal{R}_{(\alpha\mu)}\mathcal{R}_{(\beta\nu)}g^{\alpha\beta} \\
+2c_2\, l_p^2\,\mathcal{R}_{[\alpha\mu]}\mathcal{R}_{[\beta\nu]}g^{\alpha\beta} - \frac{1}{2}c_1\, l_p^2\,\mathcal{R}_{(\alpha\beta)}\mathcal{R}^{(\alpha\beta)}g_{\mu\nu} \\
-\frac{1}{2}c_2\, l_p^2\,\mathcal{R}_{[\alpha\beta]}\mathcal{R}^{[\alpha\beta]}g_{\mu\nu} = \kappa T_{\mu\nu}
\end{aligned}
$$

$$
\begin{aligned}
ii)\ \ \nabla_\lambda\left[\sqrt{-g}\left(g^{\mu\nu} + 2\,c_1\, l_p^2\mathcal{R}^{(\mu\nu)}\right)\right] \\
+\frac{2}{3}c_2\, l_p^2\,\nabla_\sigma\left[\sqrt{-g}\mathcal{R}^{[\mu\sigma]}\delta^\nu{}_\lambda + \sqrt{-g}\mathcal{R}^{[\nu\sigma]}\delta^\mu{}_\lambda\right] = 0
\end{aligned}
$$

The revenge of Palatini theories

Simple but characteristic example, set $c_1 = 0$. The field equations can be recast as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \kappa F_{\alpha\mu}F_{\beta\nu}g^{\alpha\beta} - \frac{1}{4}\kappa F_{\alpha\beta}F^{\alpha\beta}g_{\mu\nu} + \kappa m^2 A_\mu A_\nu - \frac{1}{2}\kappa m^2 A^\sigma A_\sigma g_{\mu\nu} + \kappa T_{\mu\nu}$$

$$\bar{\nabla}_\mu F^{\mu\nu} - m^2 A^\nu = 0$$

with $A_\mu = \sqrt{|c_2|/(4\pi)}\, Q_\mu$, $F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}$ and $m^2 = 3/(|c_2| l_p^2)$

The revenge of Palatini theories

Simple but characteristic example, set $c_1 = 0$. The field equations can be recast as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \kappa F_{\alpha\mu} F_{\beta\nu} g^{\alpha\beta} - \frac{1}{4}\kappa F_{\alpha\beta} F^{\alpha\beta} g_{\mu\nu} + \kappa m^2 A_\mu A_\nu - \frac{1}{2}\kappa m^2 A^\sigma A_\sigma g_{\mu\nu} + \kappa T_{\mu\nu}$$

$$\bar{\nabla}_\mu F^{\mu\nu} - m^2 A^\nu = 0$$

with $A_\mu = \sqrt{|c_2|/(4\pi)}\, Q_\mu$, $F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]}$ and $m^2 = 3/(|c_2| l_p^2)$

The theory is dynamically equivalent to a theory with $S_{\rm TOT} = S_{EH} + S_{EP} + S_M(\psi, g_{\mu\nu})$, where

$$S_{EP} = 8\pi \int dx^4 \sqrt{-g}\Big[-\frac{1}{2} F_{\mu\nu} F^{\mu\nu} - m^2 A^\mu A_\mu \Big]$$

Metric-affine theories in a nutshell

More general framework of modified gravities: metric-affine theories. Simplest MAG: the lesson of the ECSK theory.

$$(L_4, g) \xrightarrow{Q=0} U_4 \xrightarrow{S=0} V_4 \xrightarrow{R=0} R_4$$

Preferred curves in Riemann-Cartan U_4:
autoparallel vs extremals curves.

Field equations:
Einstein tensor$=k*$energy momentum
torsion$=k*$spin angular momentum

Invariance of a special relativistic theory of matter under Poincaré transformation inexorably leads to U_4!

Main consequences of U_4 theory

No waves of torsion outside
the spinning matter distribution!
But gravitational waves produced
by processes involving spin...

U_4 theory predicts a new, very weak, universal spin contact interaction of gravitational origin.

Critical mass density typically huge:

$$\rho = mn, \quad s = \hbar n/2 \quad \rightarrow \overline{\rho} = \frac{m^2}{k\hbar^2}$$

To be considered in high density regimes...

Particle pair creation when the mass density reaches the critical density $\overline{\rho}$.

Coupling torsion with matter fields

Scalar field: no spin ⇒ no coupling to torsion

Maxwell and non-Abelian gauge fields:
minimally coupling to torsion ⇒ gauge symmetry breaking.

Proca field: problem of gauge non-invariance bypassed.

Dirac field:

$$\begin{aligned}\mathcal{L}_D[\Gamma] &= (\hbar c/2)[(\nabla_\alpha\overline{\psi})\gamma^\alpha\psi - \overline{\psi}\gamma^\alpha\nabla_\alpha\psi - 2(mc/\hbar)\overline{\psi}\psi] \\ &= \mathcal{L}_D[\{\}] + \mathrm{Spin}\otimes\mathrm{Torsion}\end{aligned}$$

$$\gamma^\alpha\nabla_\alpha\psi + \frac{3}{8}l_P^2(\overline{\psi}\gamma_5\gamma^\alpha\psi)\gamma_5\gamma_\alpha\psi + (mc/\hbar)\psi = 0$$

Neutrinos: Dirac with no spin contact term.

Power counting analysis

Generalization of metric-affine theories.

The action will be of the following general form

$$S = S_G + S_M = \int d^4x \sqrt{-g} \left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^\rho{}_{\mu\nu}) + \mathcal{L}_M \left(g_{\mu\nu}, \Gamma^\rho{}_{\mu\nu}, \psi \right) \right]$$

One needs to specify the exact form of the Lagrangian $\mathcal{L}_G$.

Power counting analysis

Generalization of metric-affine theories.

The action will be of the following general form

$$S = S_G + S_M = \int d^4x\sqrt{-g}\left[\mathcal{L}_G(g_{\mu\nu}, \Gamma^\rho{}_{\mu\nu}) + \mathcal{L}_M\left(g_{\mu\nu}, \Gamma^\rho{}_{\mu\nu}, \psi\right)\right]$$

One needs to specify the exact form of the Lagrangian $\mathcal{L}_G$.

We set $c = 1$ and choose the dimensions $[dx] = [dt] = [l]$.
Then we have

$$[g_{\mu\nu}] = [1]\,, [\sqrt{-g}\,dx^4] = [l^4]\,, [\Gamma^\lambda{}_{\mu\nu}] = [l^{-1}]\,, [\mathcal{R}_{\mu\nu}] = [l^{-2}]$$

Second order action

The usual $\mathcal{L}_{EH} = \mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is at second order. Which other terms can we write at this order?

Second order action

The usual $\mathcal{L}_{EH} = \mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}$ is at second order. Which other terms can we write at this order?

The Cartan torsion tensor $S_{\mu\nu}{}^{\lambda}$ has the same dimension of the connection. Therefore, due to symmetries of torsion, the only terms we can write are

$$g^{\mu\nu}\nabla_{\mu}S_{\nu\sigma}{}^{\sigma}$$

$$g^{\mu\nu}S_{\mu\lambda}{}^{\lambda}S_{\nu\sigma}{}^{\sigma}, \qquad g^{\mu\nu}S_{\mu\lambda}{}^{\sigma}S_{\nu\sigma}{}^{\lambda}, \qquad g^{\mu\alpha}g^{\nu\beta}g_{\lambda\gamma}S_{\mu\nu}{}^{\lambda}S_{\alpha\beta}{}^{\gamma}$$

This is not the end of the story: Levi-Civita curvature, non-metricity, $\Gamma = \{\} + C$

Field equations

You really don't want to see them... [VV, Sotiriou, Liberati 2010, 2011] Define the stress-energy tensor and the hypermomentum as

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}\,, \qquad \Delta_\lambda{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta \Gamma^\lambda{}_{\mu\nu}}$$

it is possible to show that the connection can be algebraically solved in terms of the matter fields and the metric $\Gamma \sim \{\} + f(\Delta)$ If put into metric field equation

$$G^g_{\mu\nu} = \mathcal{T}_{\mu\nu}(T_{\mu\nu}, \Delta_\lambda{}^{\mu\nu}, g_{\mu\nu})$$

Field equations

You really don't want to see them... [VV, Sotiriou, Liberati 2010, 2011] Define the stress-energy tensor and the hypermomentum as

$$T_{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta g^{\mu\nu}} , \qquad \Delta_\lambda{}^{\mu\nu} \equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_M}{\delta \Gamma^\lambda{}_{\mu\nu}}$$

it is possible to show that the connection can be algebraically solved in terms of the matter fields and the metric $\Gamma \sim \{\} + f(\Delta)$ If put into metric field equation

$$G^g_{\mu\nu} = \mathcal{T}_{\mu\nu}(T_{\mu\nu}, \Delta_\lambda{}^{\mu\nu}, g_{\mu\nu})$$

The independent connection does NOT carry any dynamics if:

- the matter action depends at most linearly on the connection (scalar and gauge fields; fermions)
- more complicated expressions involving only first order derivatives of matter fields

Hard life with further invariants...

Just a taste of what we have at the next meaningful order...

$$\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}\mathcal{R}^{\mu}{}_{\nu\lambda\sigma},\nabla_{\mu}\nabla_{\nu}\mathcal{R}^{\alpha}{}_{\beta\gamma\delta},\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}S_{\mu\nu}{}^{\lambda}S_{\tau\omega}{}^{\rho},\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}\nabla_{\rho}S_{\mu\nu}{}^{\lambda}$$
$$S_{\mu\nu}{}^{\lambda}\nabla_{\rho}\mathcal{R}^{\alpha}{}_{\beta\gamma\delta},S_{\mu\nu}{}^{\lambda}S_{\alpha\beta}{}^{\sigma}S_{\gamma\delta}{}^{\kappa}S_{\tau\omega}{}^{\rho},S_{\mu\nu}{}^{\lambda}S_{\alpha\beta}{}^{\sigma}\nabla_{\rho}S_{\gamma\delta}{}^{\kappa}$$
$$S_{\mu\nu}{}^{\lambda}\nabla_{\rho}\nabla_{\kappa}S_{\alpha\beta}{}^{\sigma},\nabla_{\rho}S_{\mu\nu}{}^{\lambda}\nabla_{\kappa}S_{\alpha\beta}{}^{\sigma},\nabla_{\mu}\nabla_{\nu}\nabla_{\rho}S_{\alpha\beta}{}^{\sigma}$$

Hard life with further invariants...

Just a taste of what we have at the next meaningful order...

$$\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}\mathcal{R}^{\mu}{}_{\nu\lambda\sigma}, \nabla_\mu\nabla_\nu\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}, \mathcal{R}^{\alpha}{}_{\beta\gamma\delta}S_{\mu\nu}{}^{\lambda}S_{\tau\omega}{}^{\rho}, \mathcal{R}^{\alpha}{}_{\beta\gamma\delta}\nabla_\rho S_{\mu\nu}{}^{\lambda}$$
$$S_{\mu\nu}{}^{\lambda}\nabla_\rho\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}, S_{\mu\nu}{}^{\lambda}S_{\alpha\beta}{}^{\sigma}S_{\gamma\delta}{}^{\kappa}S_{\tau\omega}{}^{\rho}, S_{\mu\nu}{}^{\lambda}S_{\alpha\beta}{}^{\sigma}\nabla_\rho S_{\gamma\delta}{}^{\kappa}$$
$$S_{\mu\nu}{}^{\lambda}\nabla_\rho\nabla_\kappa S_{\alpha\beta}{}^{\sigma}, \nabla_\rho S_{\mu\nu}{}^{\lambda}\nabla_\kappa S_{\alpha\beta}{}^{\sigma}, \nabla_\mu\nabla_\nu\nabla_\rho S_{\alpha\beta}{}^{\sigma}$$

In such a case, the independent connection cannot be eliminated: even if one discardes the obvious cases $(\nabla S)^2$ or S^4, an action with just the curvature invariants inevitably lead to field equations where Γs cannot be algebraically expressed in terms of matter fields and metric

$$\mathcal{L} = \mathcal{R} + a\mathcal{R}_{\mu\nu}\mathcal{R}_{\kappa\lambda}\left(c_1 g^{\mu\kappa}g^{\nu\lambda} + c_2 g^{\mu\lambda}g^{\nu\kappa}\right)$$

$f(\mathcal{R})$-gravity

A rather peculiar case

$$\mathcal{S} = \frac{1}{16\pi l_p^2}\int d^4x\sqrt{-g}\left[f(\mathcal{R}) + B^\mu S_\mu\right] + S_m$$

Exploiting field equations we get

$$\Gamma^\rho{}_{\alpha\beta} = \left\{{}^{\rho}{}_{\alpha\beta}\right\} + \frac{1}{2f'}\left[\partial_\alpha f'\delta^\rho{}_\beta + \partial_\beta f'\delta^\rho{}_\alpha - g^{\rho\lambda}g_{\alpha\beta}\partial_\lambda f'\right] + \frac{\Omega_{\alpha\beta}{}^\rho}{f'}$$

$$\mathcal{R}f'(\mathcal{R}) - 2f(\mathcal{R}) = \kappa T(\Gamma)$$

$$\Gamma^\rho{}_{[\alpha\beta]} \equiv S_{\alpha\beta}{}^\rho = g^{\rho\lambda}(\Delta_{\beta[\lambda\alpha]} + \Delta_{\alpha[\beta\lambda]} - \Delta_{\lambda[\alpha\beta]})$$

Torsion is still not dynamical

Conclusions

For the most general action one can construct with second order invariants the connection does not carry any dynamics and can always be algebraically eliminated.

Conclusions

For the most general action one can construct with second order invariants the connection does not carry any dynamics and can always be algebraically eliminated.

Including higher order terms in the action, the connection (or parts of it) becomes dynamical and so, it cannot be eliminated algebraically. The theory now propagates more degrees of freedom than general relativity.

Conclusions

For the most general action one can construct with second order invariants the connection does not carry any dynamics and can always be algebraically eliminated.

Including higher order terms in the action, the connection (or parts of it) becomes dynamical and so, it cannot be eliminated algebraically. The theory now propagates more degrees of freedom than general relativity.

In $f(\mathcal{R})$ actions, even though the connection does carry dynamics in the presence of fields coupling to it, torsion remains non-propagating. The propagating degrees of freedom reside only in the symmetric part of the connection.

What's going on

Inclusion of non-metricity terms in the action, here neglected for sake of simplicity:

$$Q_{\lambda\mu\nu} * S_{\alpha\beta}{}^{\gamma} * \delta * g * g$$
$$Q_{\lambda\mu\nu} * Q_{\gamma\alpha\beta} * g * g * g$$

What's going on

Inclusion of non-metricity terms in the action, here neglected for sake of simplicity:

$$Q_{\lambda\mu\nu} * S_{\alpha\beta}{}^{\gamma} * \delta * g * g$$
$$Q_{\lambda\mu\nu} * Q_{\gamma\alpha\beta} * g * g * g$$

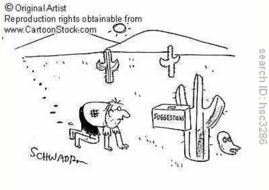
MAG vs Cosmology: spinning dark matter, Weyssenhoff fluid...

What's going on

Inclusion of non-metricity terms in the action, here neglected for sake of simplicity:

$$Q_{\lambda\mu\nu} * S_{\alpha\beta}{}^{\gamma} * \delta * g * g$$
$$Q_{\lambda\mu\nu} * Q_{\gamma\alpha\beta} * g * g * g$$

MAG vs Cosmology: spinning dark matter, Weyssenhoff fluid...



Your suggestions!!!