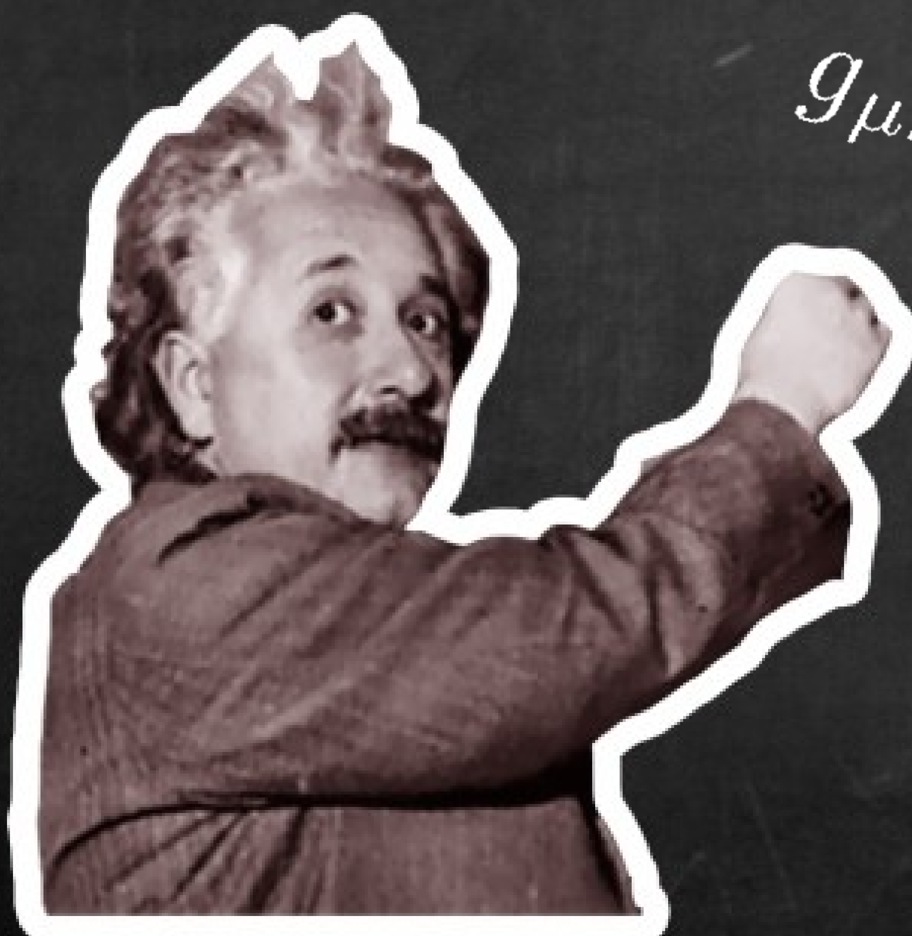
6ª Escola de

Astrofísica e Gravitação do IST

Lisboa – Septembro 4-8, 2012

Ondas Gravitacionais



$$g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu}$$

$$\Box \bar{h}_{ab} = -16\pi G T_{ab}$$

Paolo Pani

CENTRA – Instituto Superior Técnico

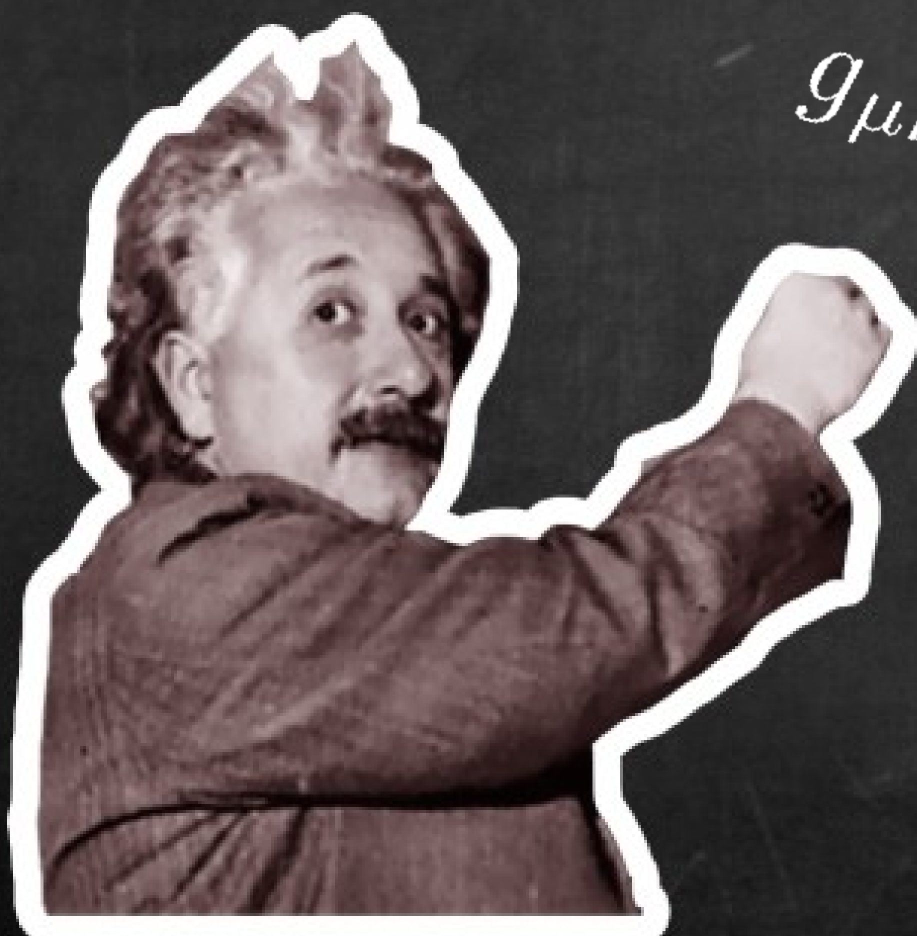
http://blackholes.ist.utl.pt

6th School in

Astrophysics and Gravitation

IST- Lisboa – September 4-8, 2012

Gravitational waves



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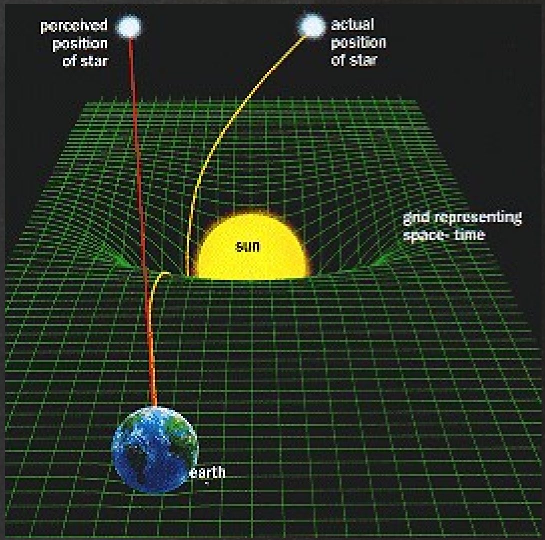
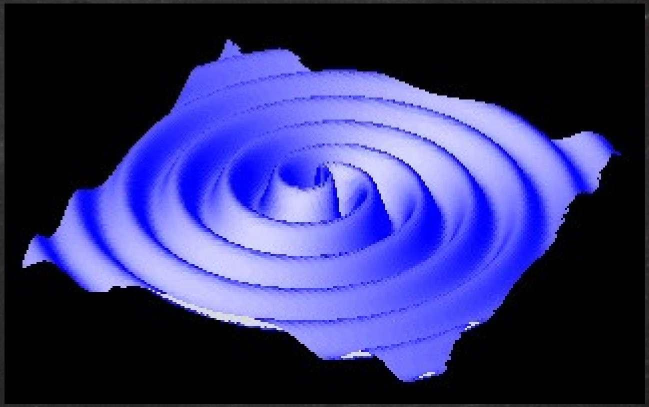
When, how, why

- Predicted by Einstein in 1916, right after GR
- Ripples in spacetime
- Cannot exist in Newtonian gravity
- Deformations from spherical symmetry
- GWs are weak! (pro and cons)



Light deflection in GR

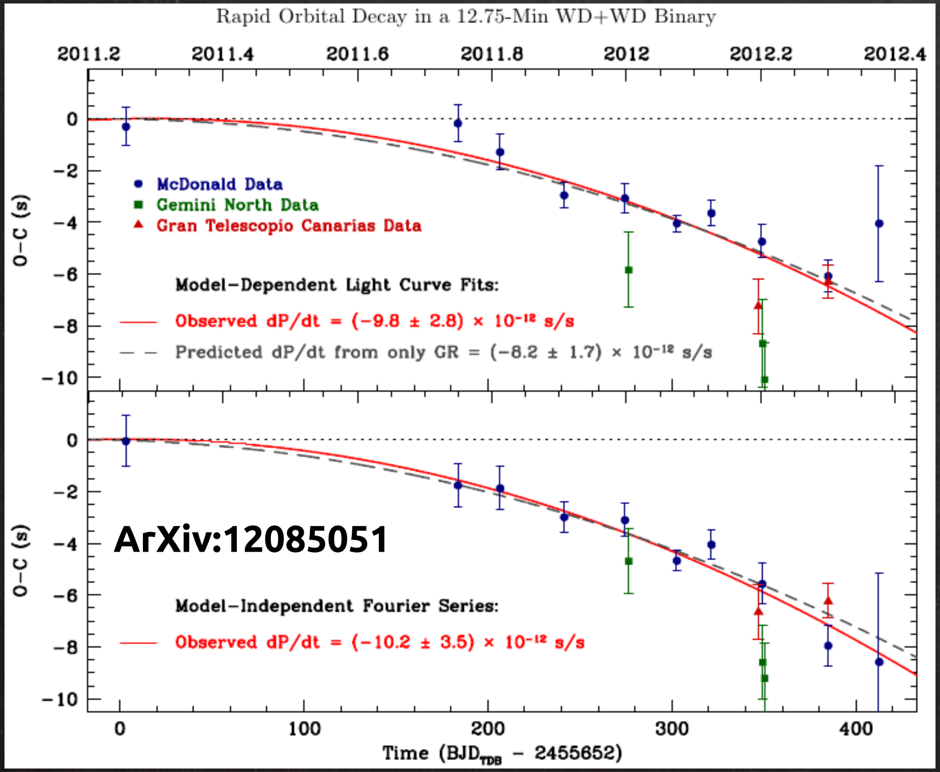


Do GWs exist?

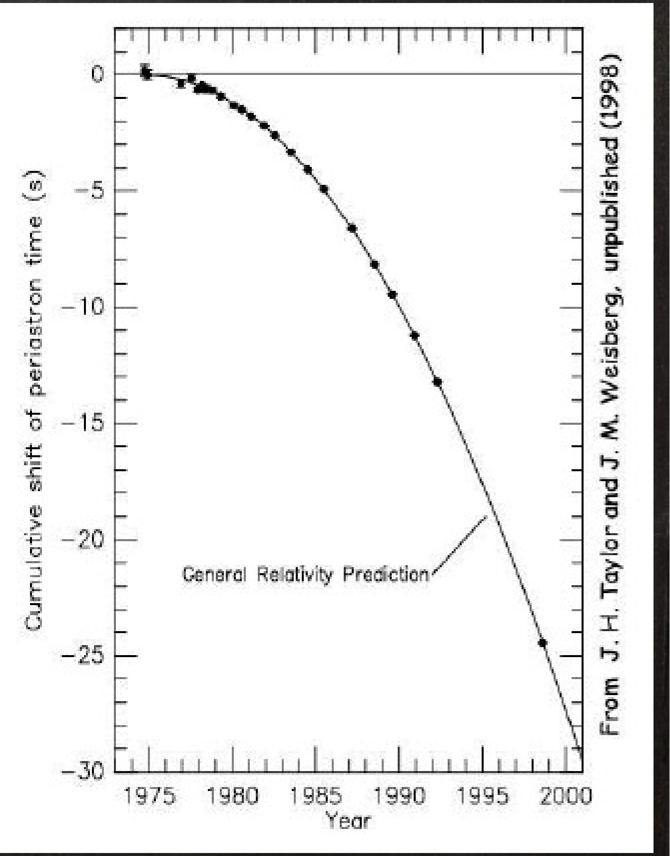
- No direct evidence yet
- Indirect evidences
- Hulse-Taylor pulsar PSR B1913+16
- WD+WD binary J0651+2844D

Weber's bar

Nobel prize 1993

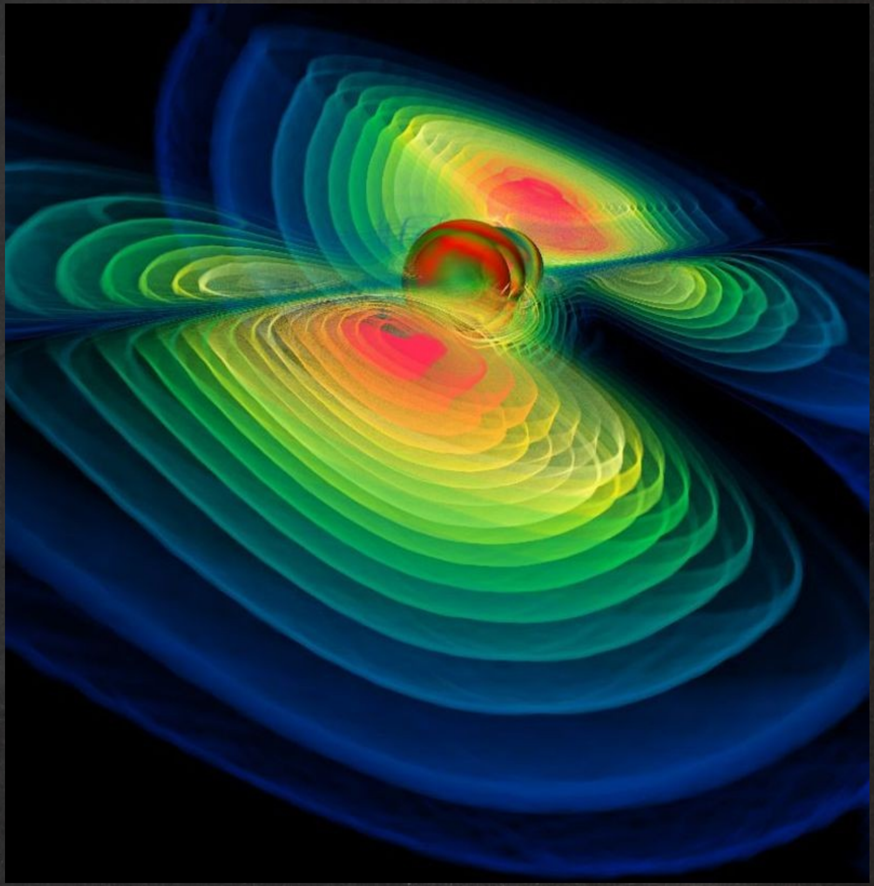


ArXiv:12085051



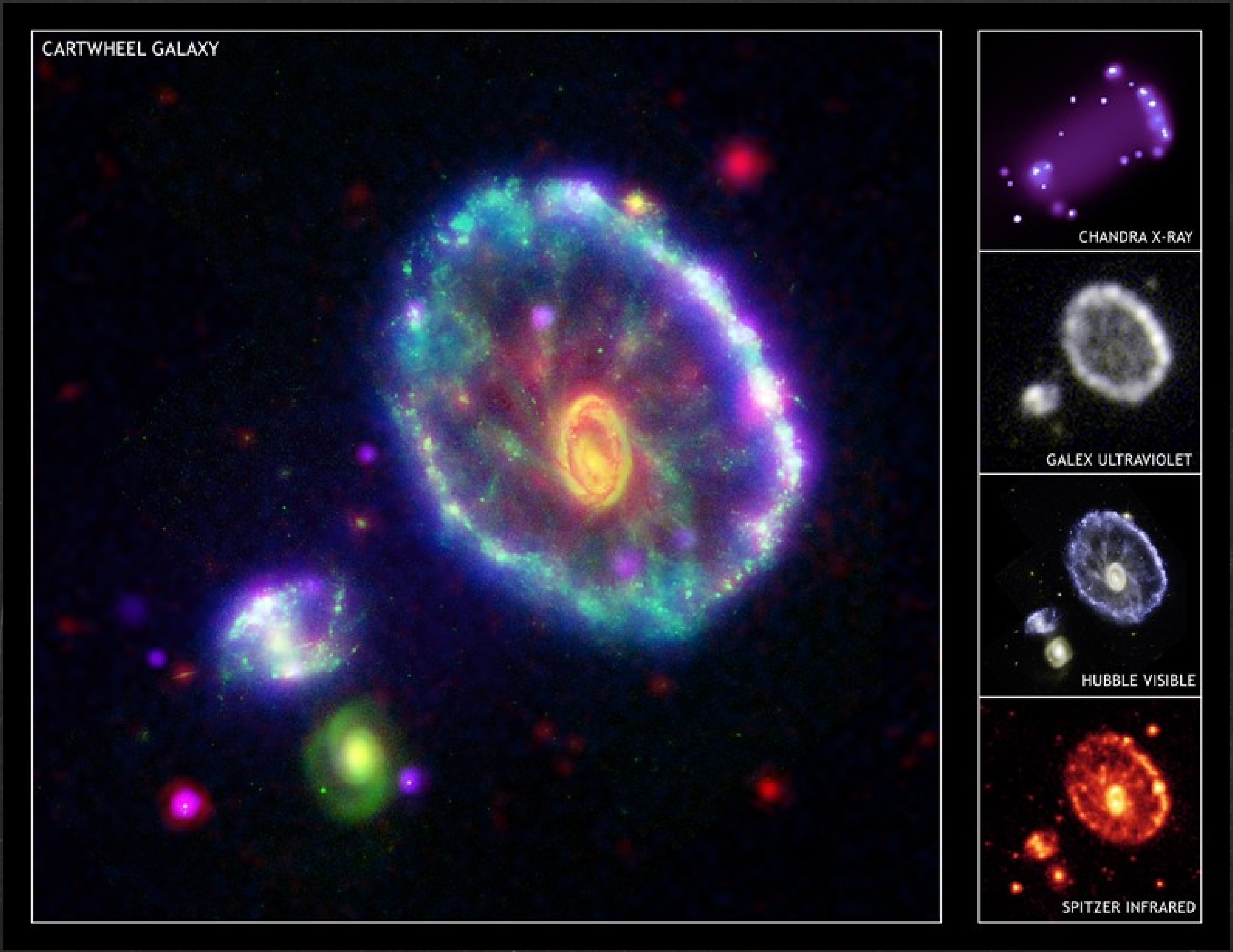
GW (astro)physics

- GW astronomy
- Tests of GR
- Big bang echoes
- Gamma-ray bursts
- Accretion disks
- Galaxy populations



Credit: MPI for Gravitational Physics/W.Benger-Z

All eyes (...and ears)



Credit: Chandra, GALEX, Hubble, Spitzer -NASA/JPL/Caltech/P.Appleton et al.

A lot of theory

Linearized GR

$$R_{ab} - \frac{1}{2} g_{ab} R + \cancel{\Lambda g_{ab}} = 8\pi G T_{ab}$$

$$g_{ab} = \eta_{ab} + \epsilon h_{ab} + \mathcal{O}(\epsilon^2)$$

$$\Gamma^a_{bc} = \epsilon \frac{\eta^{al}}{2} (h_{lc,b} + h_{lb,c} - h_{bc,a})$$

$$R_{abcd} = \frac{\epsilon}{2} (h_{ad,bc} + h_{bc,ad} - h_{ac,db} - h_{bd,ac})$$

$$R_{ab} = \eta^{cd} R_{cadb} = \frac{\epsilon}{2} (h^c_{a,bc} + h^c_{b,ac} - \Box h_{ab} - h_{,ab})$$

$$R = \eta^{ab} R_{ab} = \epsilon (h^{cd}_{,cd} - \Box h)$$

$$G_{ab} = \frac{\epsilon}{2} (h^c_{a,bc} + h^c_{b,ac} - \Box h_{ab} - h_{,ab} - \eta_{ab} h^{cd}_{,cd} + \eta_{ab} \Box h)$$

Gauge freedom

$$x^a \to x'^a = x^a + \epsilon \xi^a$$

$$h'_{ab} = h_{ab} - 2\epsilon \xi_{(b,a)}$$

Curvature tensors are gauge invariant

$$\bar{h}_{ab} \equiv h_{ab} - \frac{1}{2}\eta_{ab} h$$

$$G_{ab} = \frac{\epsilon}{2}(\bar{h}^c_{a,bc} + \bar{h}^c_{b,ac} - \Box \bar{h}_{ab} - \eta_{ab}\bar{h}^{cd}_{,cd})$$

Transverse trace-reserved gauge:

$$\bar{h}^a_{b,a} = 0 \qquad \Box \bar{h}_{ab} = -16\pi G T_{ab}$$

In vacuum:

$$\Box h_{ab} = 0 \Rightarrow \Box R_{abcd} = 0$$

Degrees of freedom

Irreducible decomposition:

$$h_{ab} = \begin{pmatrix} -2\Phi & w_i \\ w_i & h_{ij} \end{pmatrix} \qquad h_{ij} = 2s_{ij} - 2\Psi\delta_{ij}$$

$$\Box s_{ij} = 0 \Rightarrow \Box h_{ij}^{TT} = 0 \qquad h_{ij}^{TT} = C_{ij} e^{ikx}$$

Plane waves

Geodesic deviation equation:

$$\frac{D^2}{d\tau^2} S^\mu = R^\mu_{\nu\rho\sigma} u^\nu u^\rho S^\sigma \qquad \frac{D}{d\lambda} = \frac{dx^\mu}{d\lambda}\nabla_\mu$$

$$R_{\mu 00\sigma} = \frac{1}{2}\partial_0\partial_0 h_{\mu\sigma}^{TT}$$

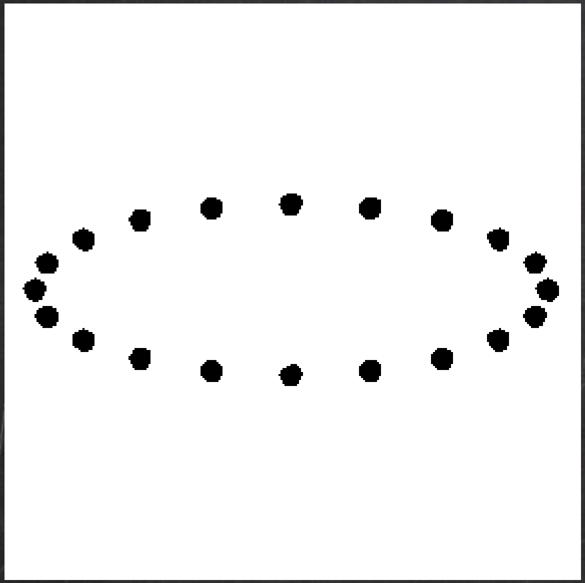
$$S^{1,2}(t) = (1 \pm \frac{1}{2} h_+ e^{ikx}) S^{1,2}(0)$$

Plus (+) polarization

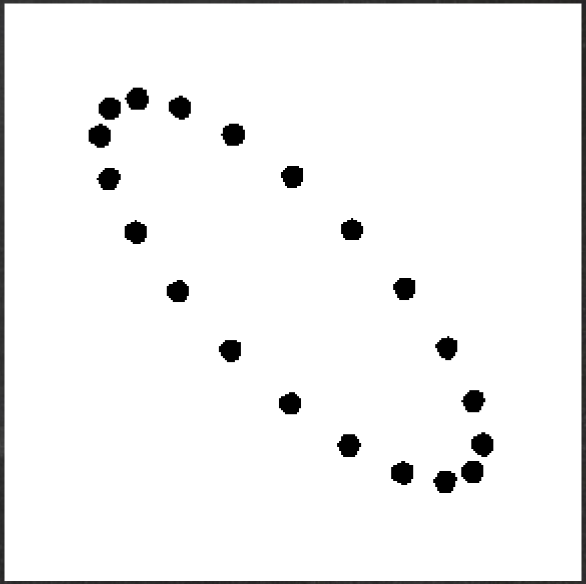
$$S^{1,2}(t) = S^{1,2}(0) + \frac{1}{2} h_x e^{ikx} S^{2,1}(0)$$

Cross (x) polarization

Degrees of freedom



Plus (+) polarization



Cross (x) polarization

~ lowest-energy modes of a string

Production of GWs (1)

$$\Box \bar{h}_{ab} = -16\pi G T_{ab}$$

Green function approach

$$\Box G(x-y) = \delta^{(4)}(x-y) \qquad \bar{h}_{ab}(x) = -16\pi G \int d^4y G(x-y) T_{ab}(y)$$

Retarded Green function

$$G(x-y) = -\frac{1}{4\pi|\bar{x}-\bar{y}|}\delta[|\bar{x}-\bar{y}| - (x^0-y^0)]\theta(x^0-y^0)$$

$$\bar{h}_{ab}(t,\bar{x}) = 4G \int d^3y \frac{T_{ab}(t_R,\bar{y})}{|\bar{x}-\bar{y}|} \qquad t_R \equiv t - |\bar{x}-\bar{y}|$$

Fourier transform

$$\tilde{\bar{h}}_{ab}(\omega,\bar{x}) = 4G \int d^3y \frac{e^{i\omega|\bar{x}-\bar{y}|}}{|\bar{x}-\bar{y}|}\tilde{T}_{ab}(t_R,\bar{y})$$

Production of GWs (2)

$$\tilde{\bar{h}}_{ab}(\omega, \bar{x}) = 4G \int d^3y \frac{e^{i\omega|\bar{x}-\bar{y}|}}{|\bar{x}-\bar{y}|} \tilde{T}_{ab}(t_R, \bar{y})$$

Isolated source, far away from the observer and slowly moving

$$\frac{e^{i\omega|\bar{x}-\bar{y}|}}{|\bar{x}-\bar{y}|} \approx \frac{e^{i\omega r}}{r}$$

Quadrupole formula (Lorenz gauge)

$$\bar{h}_{ij}(t, \bar{x}) = \frac{2G}{r} \left. \frac{d^2[I_{ij}]}{dt^2} \right|_{t_R = t-r}$$

$$I_{ij}(t) = \int d^3y y_i y_j T^{00}$$

Quadrupole moment tensor

- **No dipole (unlike in EM): conservation of the total momentum**
- **GWs are weak!**
- **GW production is coherent**

Production of GWs (3)

Gravitational radiation emitted by a binary star:

$$\bar{h}_{ij}(t,\bar{x}) \propto \frac{8GM}{r}\Omega^2 R^2$$

Massive object

Large velocity

Small distance from us

Energy loss in GWs

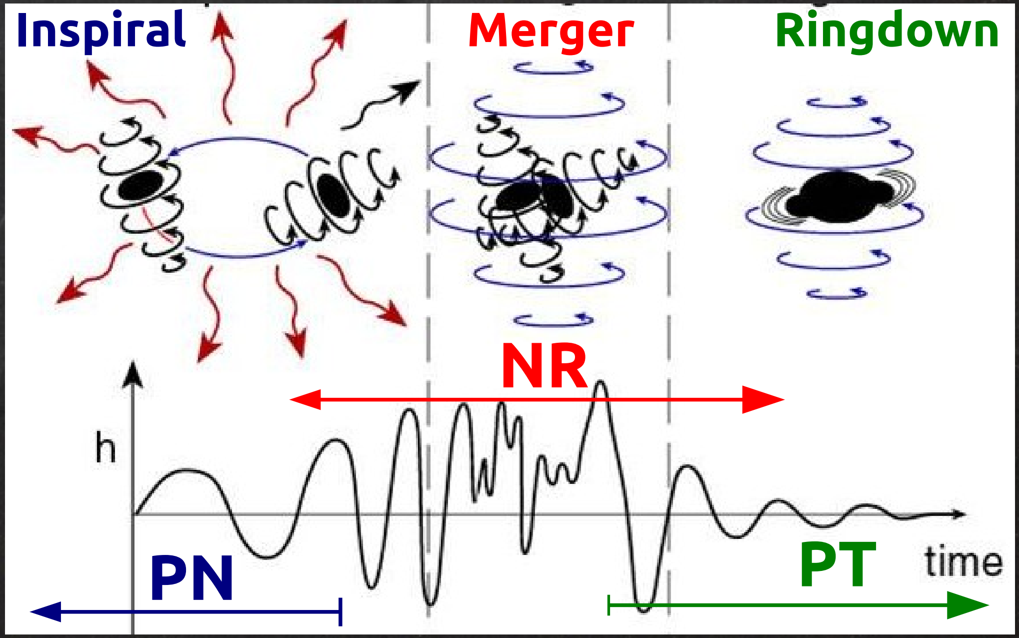
$$P = -\frac{32}{5}\frac{G^4 M^3 \mu^2}{c^5 R^5}$$

Quadrupole formula

In scalar-tensor theories:

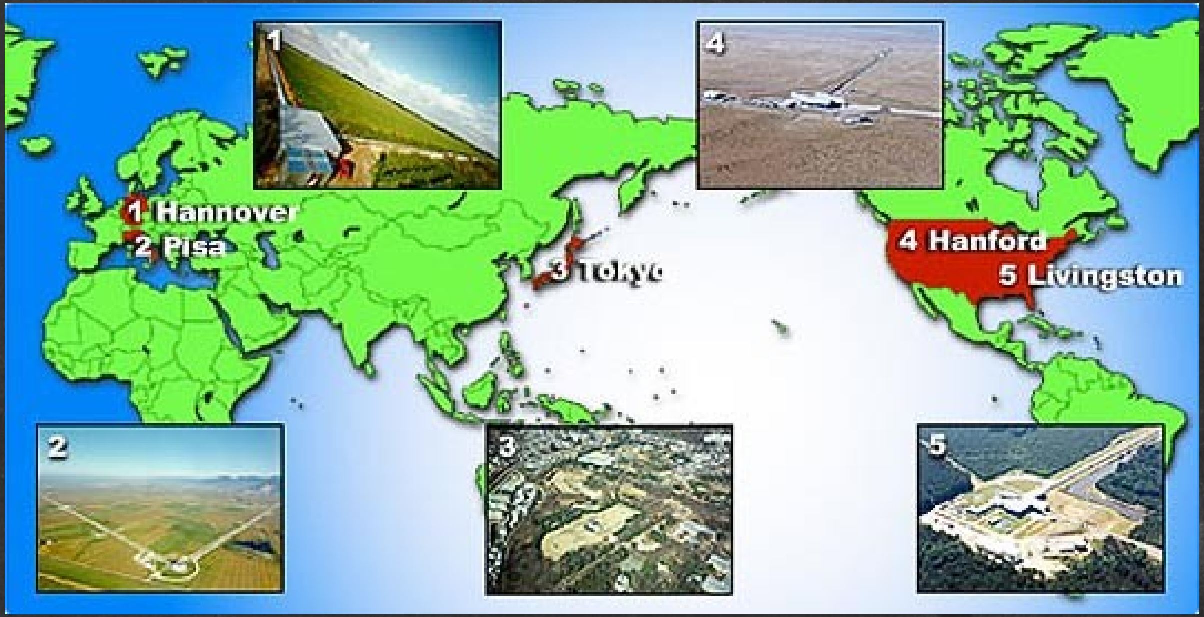
$$P = -\frac{2}{3}\frac{\mu^2 M^2}{R^4}\frac{\mathcal{S}^2}{\omega_{\rm BD}}$$

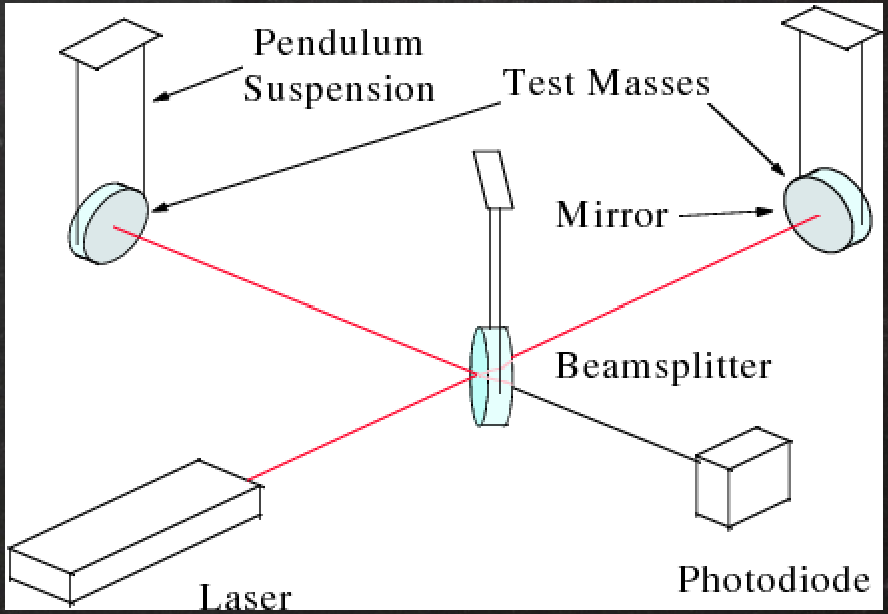
Dipole formula



Adapted from Thorne

GW detectors





Detection of GWs

$$f = \frac{\Omega}{2\pi} = \frac{c R_S^{1/2}}{4\pi\sqrt{2} R^{3/2}}$$

frequency

$$\Omega = \left(\frac{GM}{4R^3}\right)^{1/2}$$

from Kepler's law

$$R_S = \frac{2GM}{c^2}$$

Schwarzschild radius

$$\bar{h}_{ij}(t, \bar{x}) \propto \frac{8GM}{r} \Omega^2 R^2 \Rightarrow h \sim \frac{R_S^2}{rR}$$

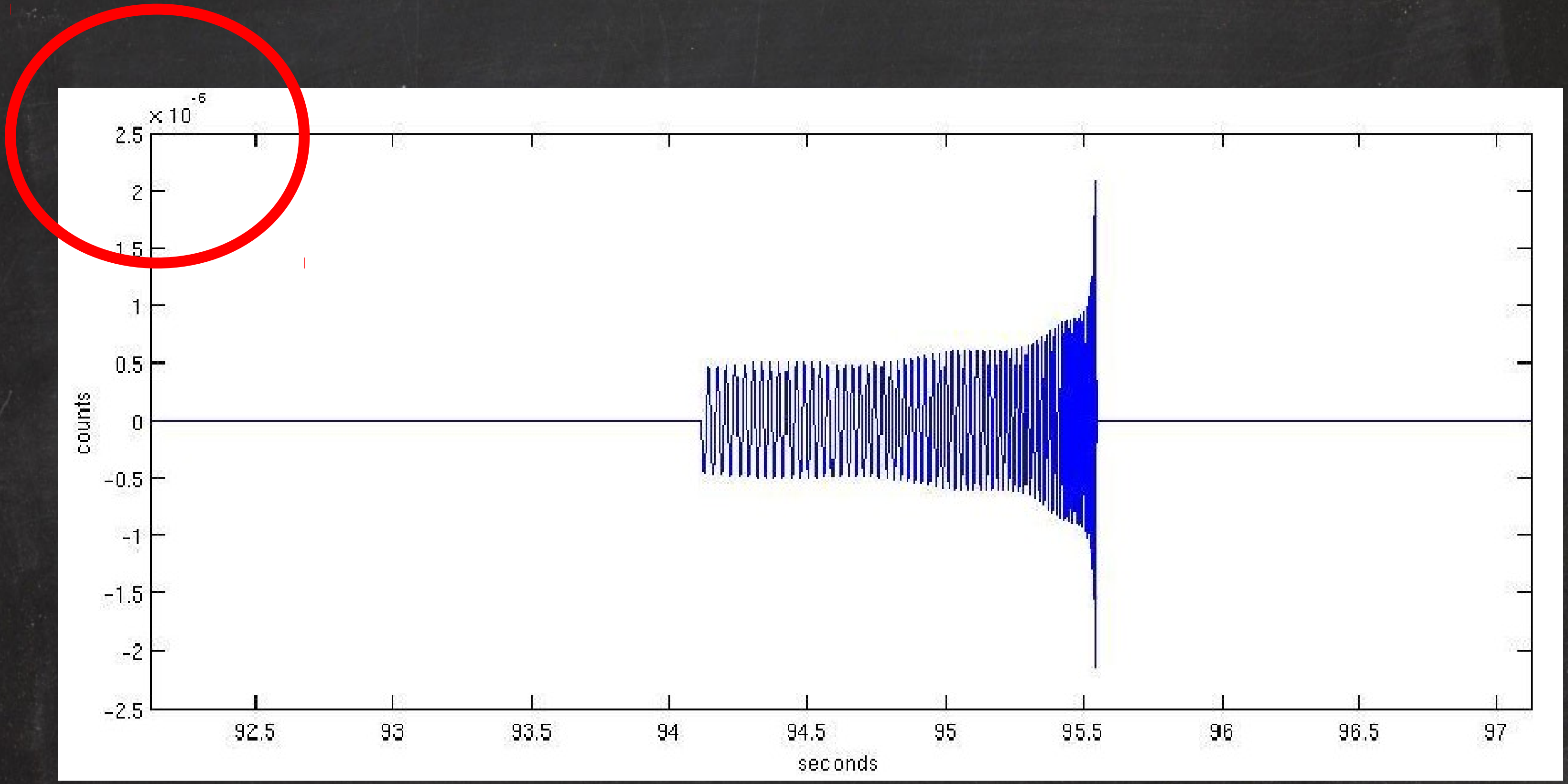
$$h \sim \frac{\delta L}{L}$$

sensitivity

$$M \sim 10 M_\odot\,, \quad R \sim 10 R_S\,, \quad r \sim 100\mathrm{Mpc} \Rightarrow \begin{cases} f \sim 10^2\mathrm{Hz} \\ h \sim 10^{-21} \end{cases}$$

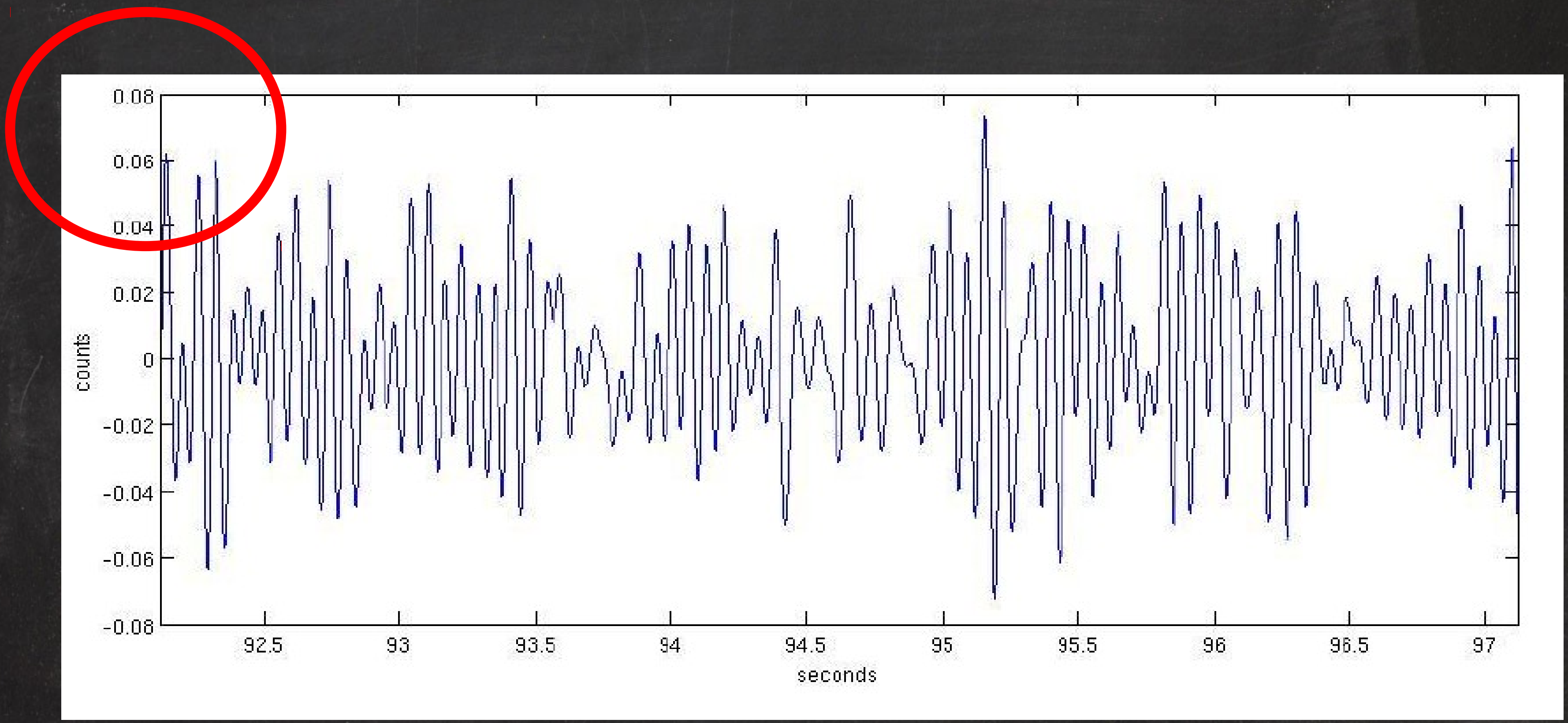
$$\delta L \sim 10^{-3} \frac{h}{10^{-21}} \frac{L}{\mathrm{km}} \mathrm{fm}$$

Detection of GWs



What you would like to see

Detection of GWs

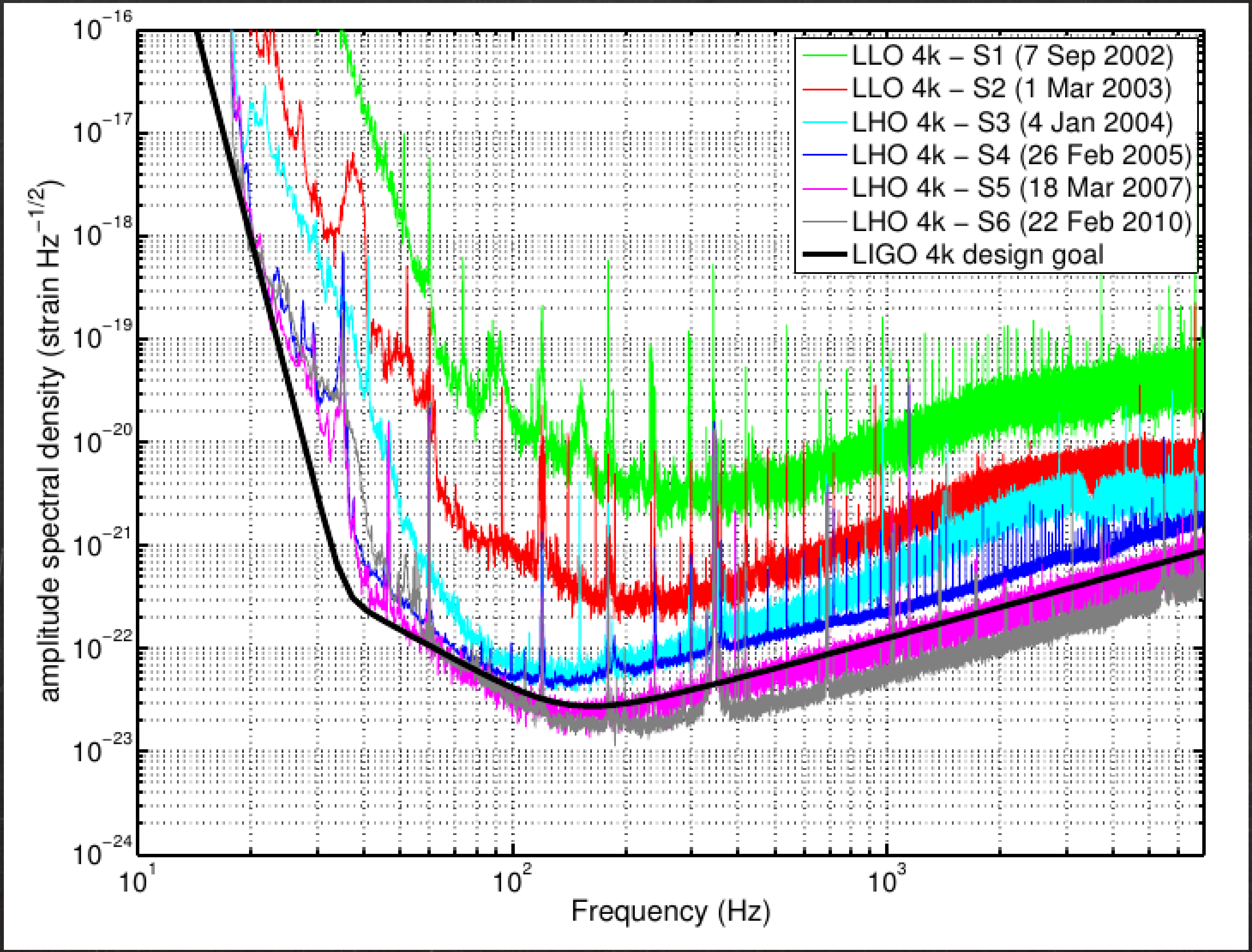


What you actually see

Extremely sensitive detector and

theoretical knowledge of the waveform

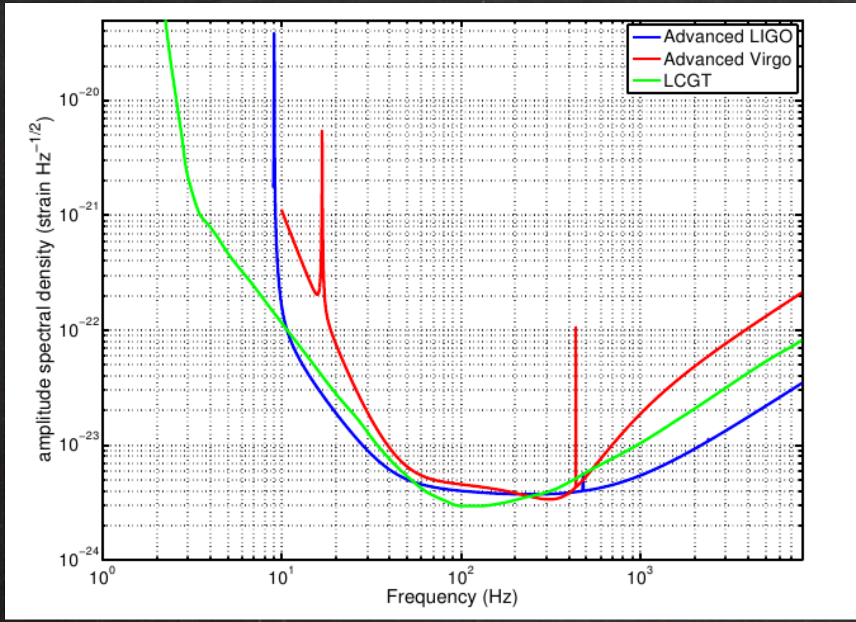
Sensitivity

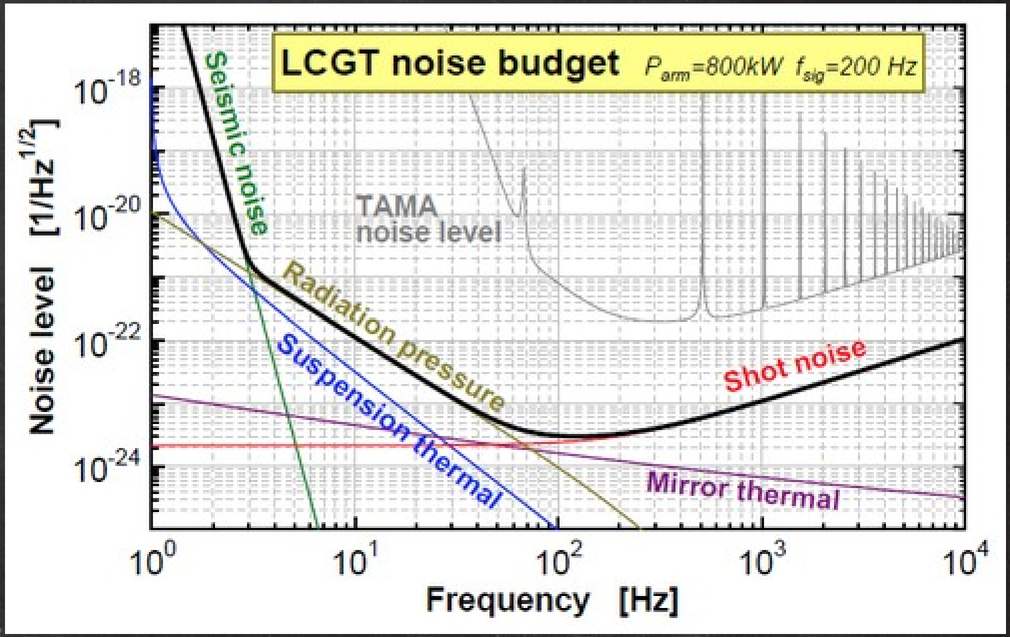


LIGO sensitivity

Planes, trains, cars, earthquakes, ...

Second generation





From 0.1 (at most!) to 40 NS-NS coalescences per yr

Future (?)

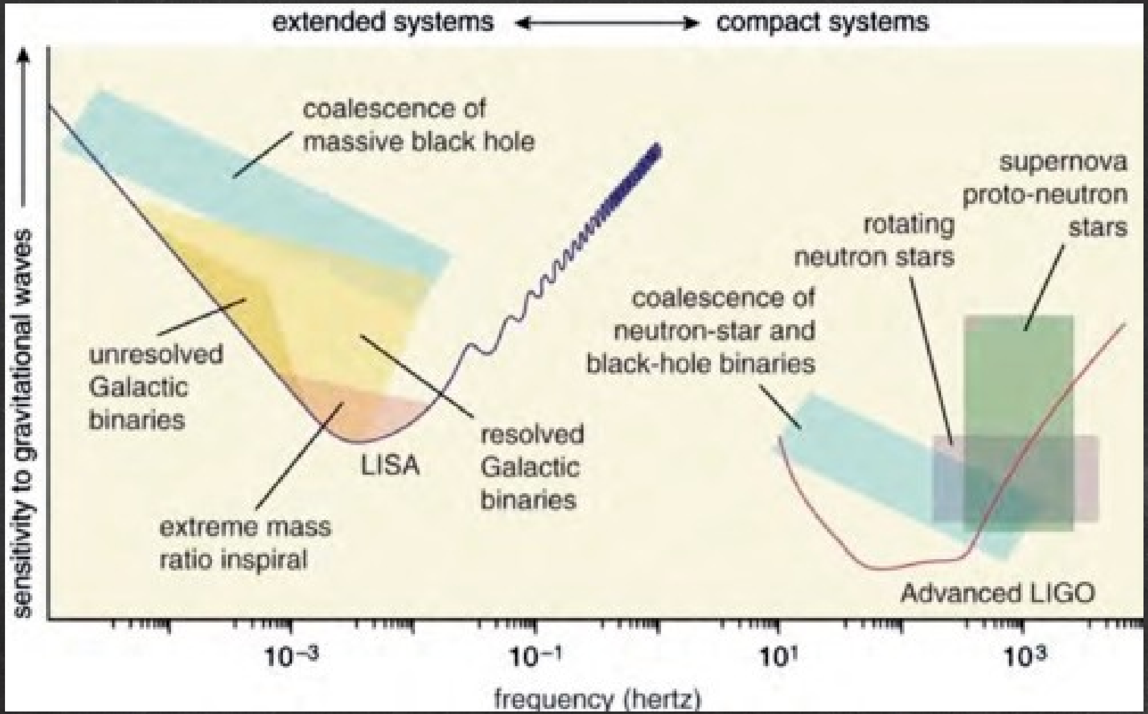
Space-based

SMBHs

Accretion

EMRIs

Stochastic bkg



Earth-based

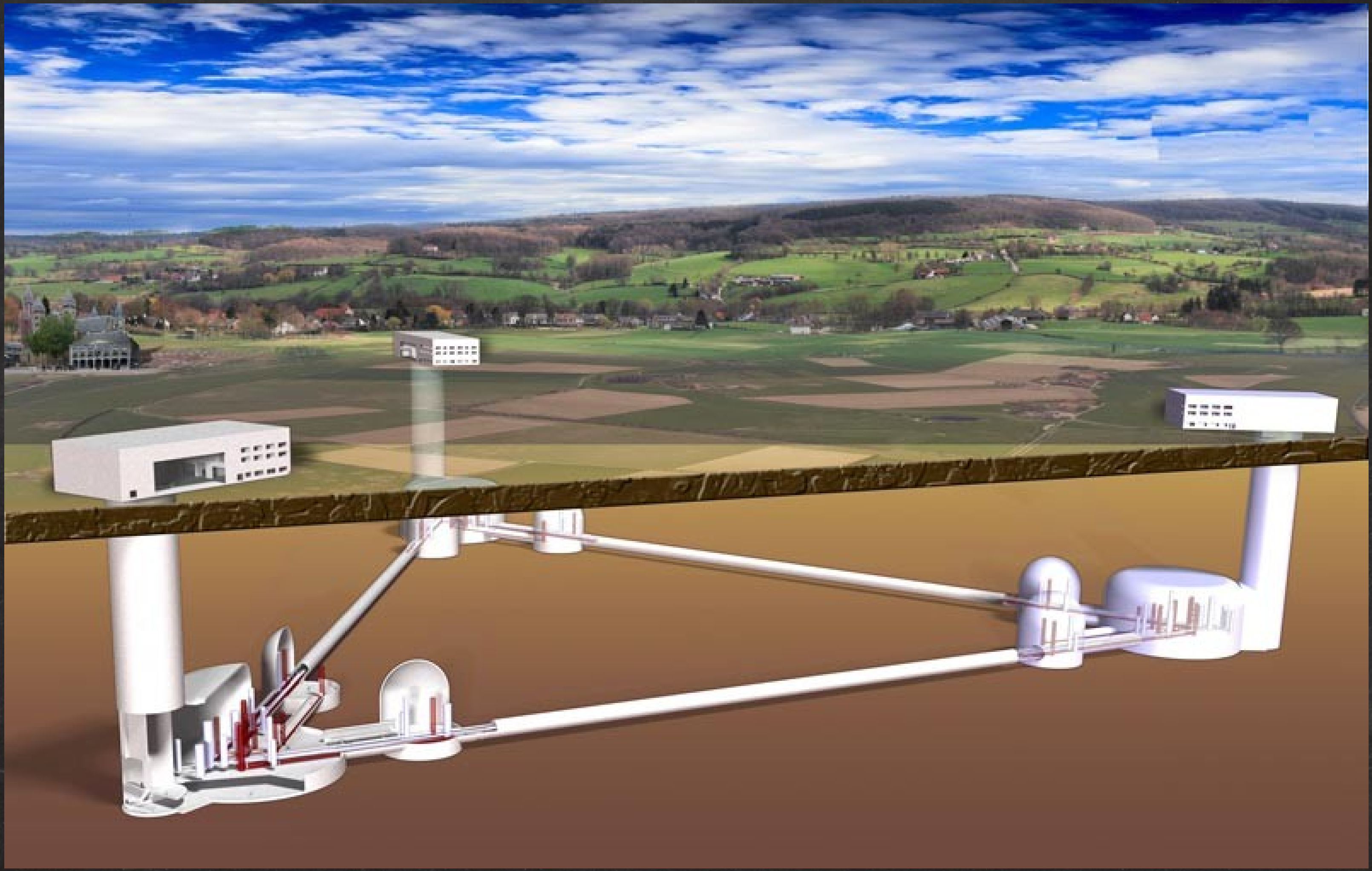
Coalescence

NS, BHs

SN collapse

Rotating NSs (?)

Future (?)



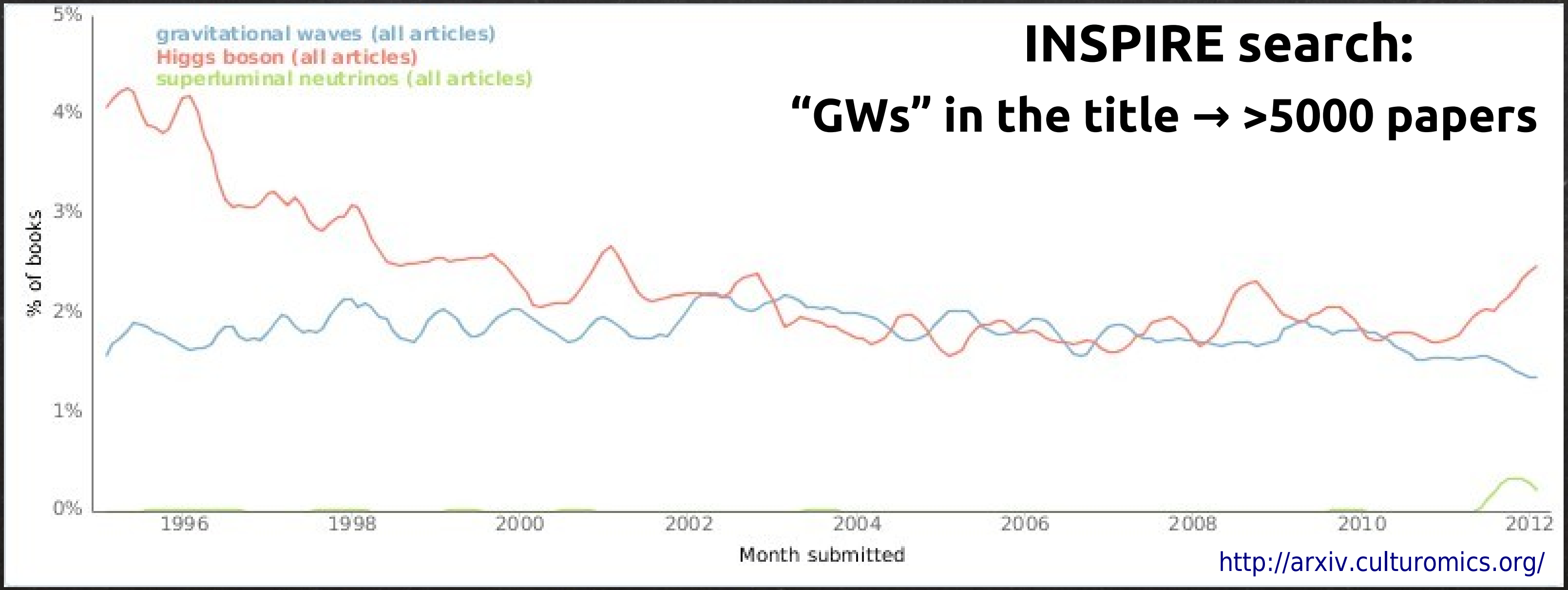
Einstein telescope ?

Conclusions

- GWs do exist!
- Their direct discovery will be one of the breakthroughs of this century
- Utmost importance for (astro)physics
- Tests of GR
- New era in astrophysics → GW astronomy
- New look at the whole universe

Some reference

- Textbooks: Hartle, MTW, D'Inverno, Carroll
- Reviews: Schutz & Ricci (gr-qc/1005.4735), Blair et al. (2012)
- LIGO reviews: Science 256 (1992) 325-333 and arXiv:0711.3041
- eLISA/NGO review: Amaro-Seoane et al. (arXiv:1201.3621)
- GW tests of GR: Will's review (gr-qc/0510072)



Want to detect GWs?

Join Einstein@home!

http://einstein.phys.uwm.edu

This work was supported by **FCT - Portugal** through PTDC projects **FIS/098025/2008, FIS/098032/2008**, **CERN/FP/123593/2011** and by the **European Community** through the **Intra-European Marie Curie contract aStronGR-2011-298297.**

MARIE CURIE ACTIONS

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Thanks!