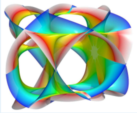
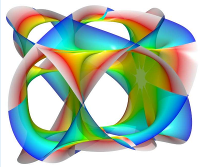
(NEW) BORDERS FOR QUANTUM COSMOLOGY

Standpoint and (some) SUSY

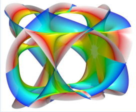
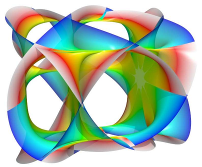
LNP 803 - 804, Springer, 2010

Paulo Vargas Moniz[1]

(1) UBI & CENTRA-IST,

... an Abstract:

Abstract

Quantum Cosmology tackles the quantum description of the early universe. It is aimed as an accessible primer that covers the basics, critically discussing ideas and concepts that comprise our current knowledge. The scope for analyzing quantum cosmological models within a supersymmetric framework is *pointed*.

As much as possible, it summarizes *what we know, what we think we know and what we think we do not know* on an equal footing. It is focused for ‘young’, inquisitive minds eager to embark on in-depth research in this field. It is hoped to suggest the tools researchers will need to go on their own, pushing them to ask the right questions rather than seek definitive answers.

Lectures

- Motif ("... *reasonably long* ...")

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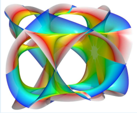
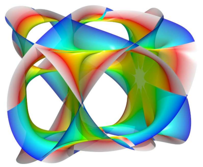
- Cases and Results ("... *reasonably accessible*...")
- Outlook (or not so):
  - Just SUSY (it)!
  - ... views on routes *yet* unexplored!

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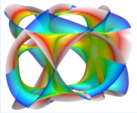
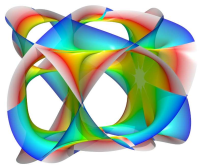

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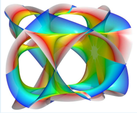
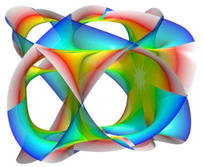

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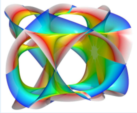
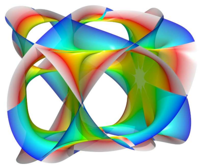

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- $\rightsquigarrow$ evolution requires a *choice* of initial conditions

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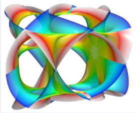
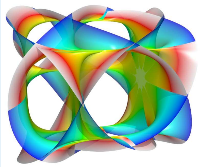

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- Challenge in the XXIth century

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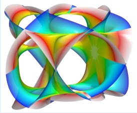
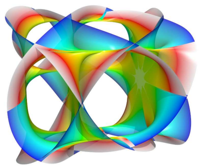

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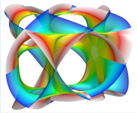

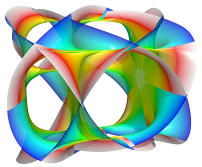

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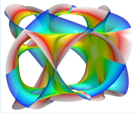

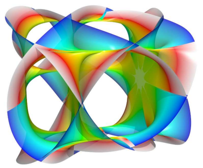

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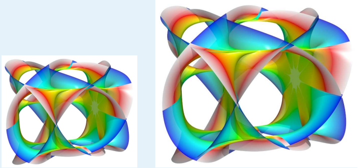

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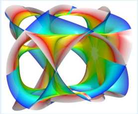
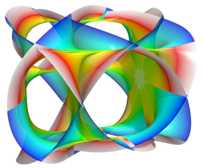

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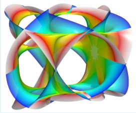
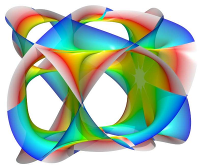

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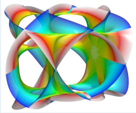
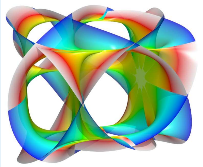

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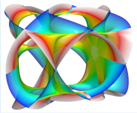
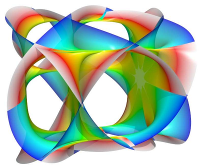

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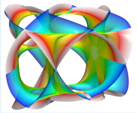 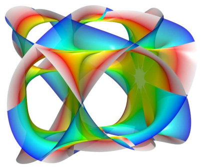

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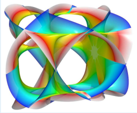
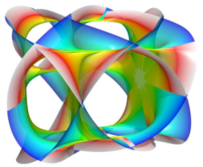
- $\rightsquigarrow$ *Primary* constraints



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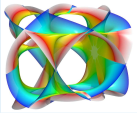
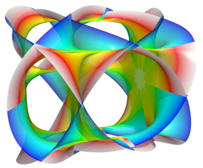
$$\pi^{ij} = -\frac{\sqrt{h}}{2k^2}\left(K^{(ij)} - h^{ij}K\right),\ \pi_\phi = \frac{\sqrt{h}}{\mathcal{N}}\left(\dot{\phi} - \mathcal{N}^i\phi_{,i}\right).$$

- $\rightsquigarrow$ *Primary* constraints
- Subsequently,
- $$S = \int dt d^3x \left(\pi^{ij}\dot{h}_{ij} + \pi_\phi\dot{\phi} - \mathcal{N}\mathcal{H}_\perp - \mathcal{N}^i\mathcal{H}_i\right),$$

Settings

- $$\begin{aligned} H &\equiv \int d^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \pi^{ij} \dot{h}_{ij} + \pi_\phi \dot{\phi} \right) - L \\ &= \int d^3x \left(\pi^0 \dot{\mathcal{N}} + \pi^i \dot{\mathcal{N}}_i + \mathcal{N} \mathcal{H}_\perp + \mathcal{N}_i \mathcal{H}^i \right) , \end{aligned} \qquad (1)$$

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$$\mathcal{H}_\perp \equiv 2k^2 \mathcal{G}_{ijkl}\pi^{ij}\pi^{kl} - \frac{\sqrt{h}}{2k^2}\left({}^{(3)}R - 2\Lambda\right) + \mathcal{H}_\perp^{\text{matter}}$$
$$= 2k^2 h^{-\frac{1}{2}}\left(\pi^{ij}\pi_{ij} - \frac{1}{2}\pi^2\right) - \frac{h^{\frac{1}{2}}\,{}^{(3)}R}{2k^2}$$
$$+ \frac{1}{2}\sqrt{h}\left[\frac{\pi_\phi^2}{h} + h^{ij}\phi_{,i}\phi_{,j} + 2V\right], \qquad (2)$$
$$\mathcal{H}^i \equiv -2\pi^{ij}{}_{|j} + h^{ij}\phi_{,j}\pi_\phi, \qquad (3)$$

Settings

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- Take notice:

$$\mathcal{G}_{ijkl} = \frac{1}{2} h^{-1/2} \left(h_{ik} h_{jl} + h_{il} h_{jk} - h_{ij} h_{kl} \right)$$

is the *DeWitt metric*; But of ‘what’?

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Settings

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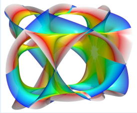
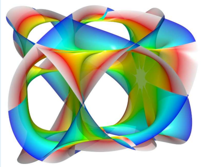

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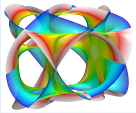
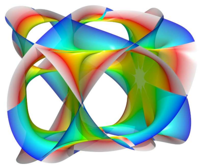

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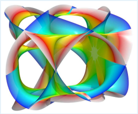
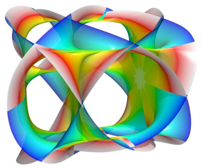

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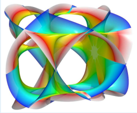
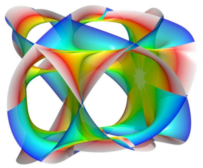

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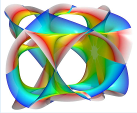
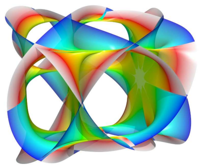

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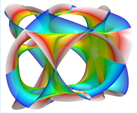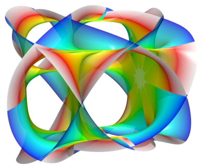

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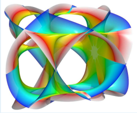
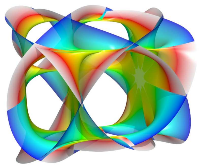

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$$\pi^{ij} \to -i\frac{\delta}{\delta h_{ij}},\ \pi_\phi \to -i\frac{\delta}{\delta \phi},\ \pi^0 \to -i\frac{\delta}{\delta \mathcal{N}},\ \pi_i \to -i\frac{\delta}{\delta \mathcal{N}_i}$$

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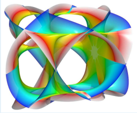 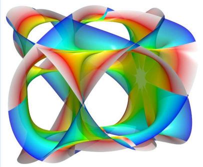

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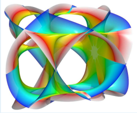
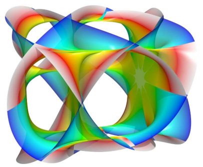

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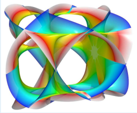
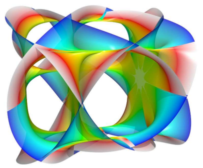

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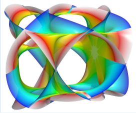
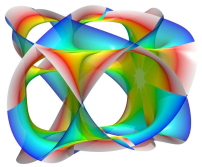

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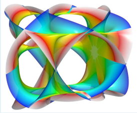
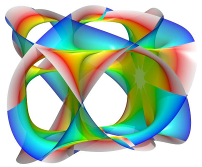

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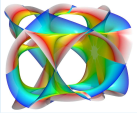
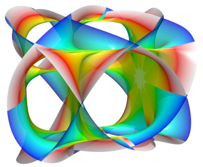

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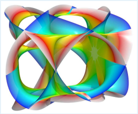 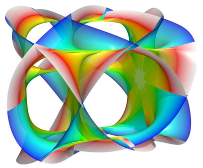

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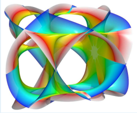
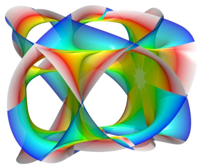

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- $\rightsquigarrow$ gauge theory
- $\rightsquigarrow$ secondary constraints
- $\rightsquigarrow$ first class constraints
- A set of generators for gauge transformations
- **Displays the symmetry properties of the system**
- SUGRA: preparation; elements to introduce and explore
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- basis *tetrad* altogether, such that $n^\mu n_\mu = -1$, $n_\mu e^\mu{}_i = 0$, $e^\mu{}_i e_{\mu j} = h_{ij}$,

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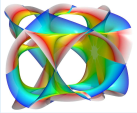
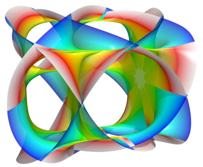

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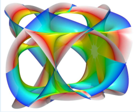

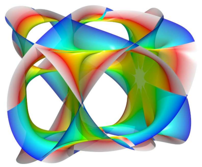

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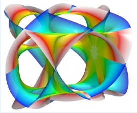 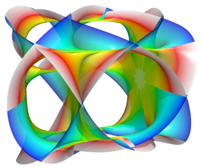

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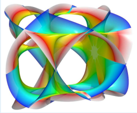
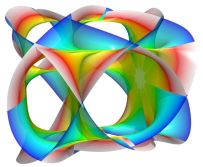

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- ... assuming path independence:

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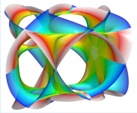
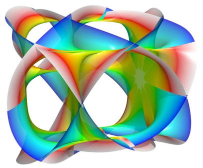
- Such theory with canonical variables $\left(h_{ij}, \pi^{ij}\right)$ satisfying

$$\left[h_{ij}(x), \pi^{kl}(x')\right] = \frac{1}{2}\left(\delta^k{}_i\delta^l{}_j + \delta^k{}_j\delta^l{}_i\right),$$

is indeed ... **General Relativity**;

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- General Relativity is thus retrieved as the theory for the metric field in a Riemannian space-time!

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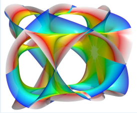
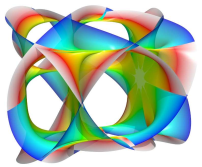

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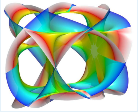
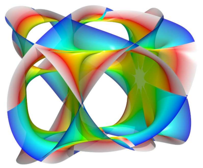

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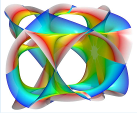
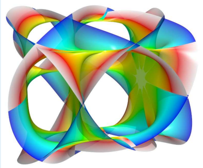

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- An intriguing route to explore:
- *proceed from the set of such (deformation) generators and their algebra to retrieve the* (yet not totally) charted *geometry associated with SUGRA, where both the metric* and *the gravitino fields are of equivalent influence*

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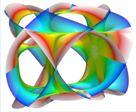
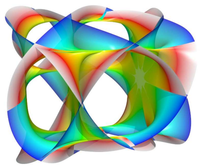

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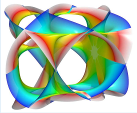
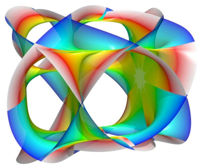
- *The amplitude to proceed from one state, e.g.,* $|h_{ij}, \varphi; \Sigma_t\rangle$, *describing the system with an intrinsic* 3*-metric,* h_{ij}*, matter configuration,* φ*, on an initial hypersurface,* Σ_t*, to another state* $|h'_{ij}, \phi^{'}; \Sigma^{'}_{t'}\rangle$*, is given by a* functional *integral of* e^{iS} *over all* 4*-geometries,* $g_{\mu\nu}$*, and matter configurations,* ϕ*, which interpolate between the initial and final configurations:*

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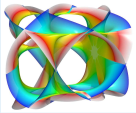
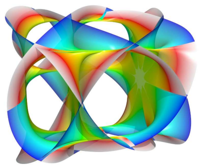

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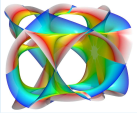

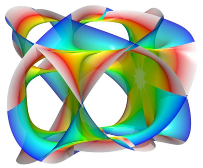

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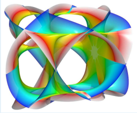
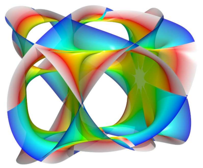

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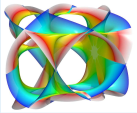
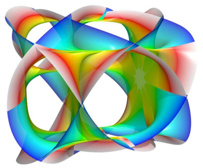
- Proposal (J Hartle , S Hawking): to restrict $\mathcal{M}$ - *only* compact Riemannian 4-manifolds

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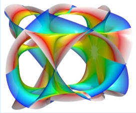
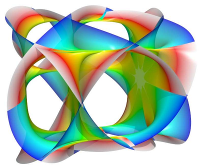

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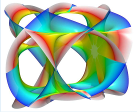 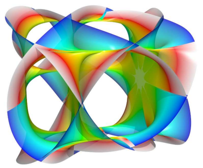

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- In other words, the boundary conditions are at the *classical* level. They correspond to
  1. The four-geometry is closed;
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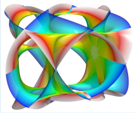
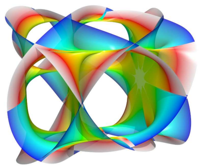

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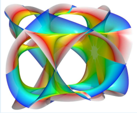
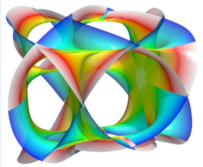

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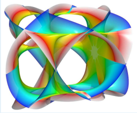
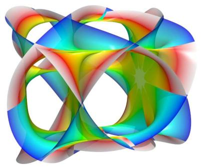

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- That is *the* challenge!

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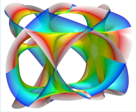

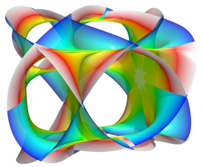

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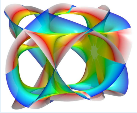
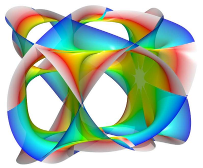

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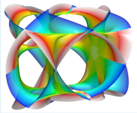
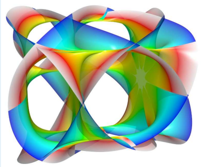
$$\mathcal{H}\Psi = \left[-\frac{1}{2}\nabla^2 + U(q)\right]\Psi = 0,$$

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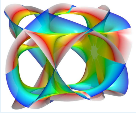
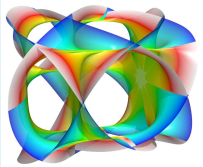
- ... a choice of *factor ordering*
- Another particular choice: *conformally invariant*

$$\mathcal{H}\Psi \equiv \left[-\frac{1}{2}\nabla^2 + \frac{\mathbf{D}-2}{8(\mathbf{D}-1)}\mathbf{R} + U(q)\right]\Psi = 0$$

Case Studies

Cases and Results

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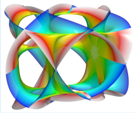 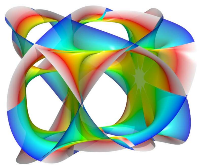

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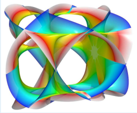
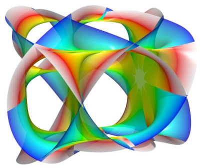

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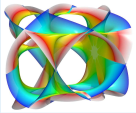
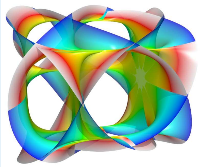

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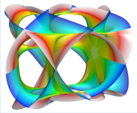
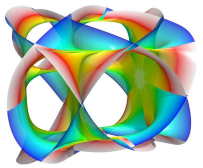
- (first-order) WKB wavefunction

$$\Psi_n = C_n \exp\left(iS_n - \frac{1}{2}\int ds\, \nabla^2 S_n\right)$$

Cases and Results

- Minisuperspaces and Boundary Conditions:





Cases and Results

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$$\mathcal{H}\Psi = \frac{1}{2}e^{-3\alpha}\left[\frac{\partial^2}{\partial\alpha^2} - \frac{\partial^2}{\partial\phi^2} - ke^{4\alpha} + e^{6\alpha}V(\phi)\right]\Psi = 0.$$

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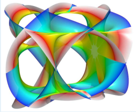
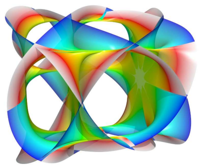
$$\Psi(a,\phi) \simeq \begin{cases} \frac{\mathcal{B}(\phi)}{a(a^2V(\phi)-k)^{1/4}} \exp\left[\frac{\pm i}{3V(\phi)}\left(a^2V(\phi) - k\right)^{3/2}\right], & a^2V > k \\ \frac{\mathcal{C}(\phi)}{a(k-a^2V(\phi))^{1/4}} \exp\left[\frac{\pm 1}{3V(\phi)}\left(k - a^2V(\phi)\right)^{3/2}\right], & a^2V < k \end{cases} .$$

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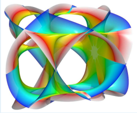
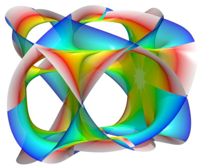
- *If V is positive, then oscillatory solutions ($\Psi \sim e^{iS}$, S being a solution of the Hamilton–Jacobi equation; $a^2V \gg |k|$, $S \simeq \pm\frac{1}{3}a^3\sqrt{V}$) will exist for large values of the scale factor, while exponential type solutions will correspond to small values of the scale factor.*

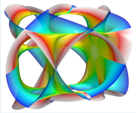
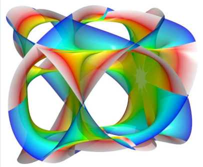
Cases and Results

- Note that then

$$\pi_\alpha = \frac{\partial S}{\partial \alpha} \Rightarrow \dot{\alpha} \simeq \pm\sqrt{V}, \quad (1)$$

$$\pi_\phi = \frac{\partial S}{\partial \phi} \Rightarrow \dot{\phi} \simeq 0, \quad (2)$$

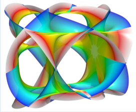
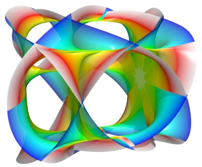
corresponding to an inflationary attractor point; i.e., ... ‘initial’ conditions *imported...!* .

Cases and Results

- The Hartle–Hawking Boundary Condition

$$\Psi_{HH}[a, \phi] = \int^{(a,\phi)} \mathcal{D}a \, \mathcal{D}\phi \, \mathcal{D}\mathcal{N} \, e^{-I[a,\phi,\mathcal{N}]},$$

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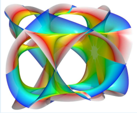
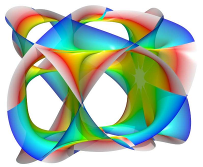

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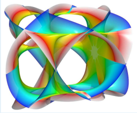
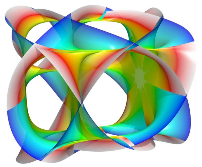

Cases and Results

- The Hartle–Hawking Boundary Condition

$$\Psi_{HH}[a,\phi] = \int^{(a,\phi)} \mathcal{D}a\, \mathcal{D}\phi\, \mathcal{D}\mathcal{N}\, e^{-I[a,\phi,\mathcal{N}]},$$

- where

$$I = \frac{1}{2}\int_0^{\tau_f} d\tau \mathcal{N} \left[\frac{-a}{\mathcal{N}^2}\left(\frac{da}{d\tau}\right)^2 + \frac{a^3}{\mathcal{N}^2}\left(\frac{d\phi}{d\tau}\right)^2 - a + a^3 V \right],$$

- *class of paths* $a(\tau_f) = a,\ \phi(\tau_f) = \phi$ *origin of the Euclidean time coordinate,* τ*, has been chosen to be zero*
- *a saddle-point approximation to the path integral taken*
- For large a (with $a^2 V \ll 1$),

$$\Psi \sim \frac{1}{\sqrt{\pi}\, a} \exp\left(\frac{1}{2}a^2\right)\left[1 + \mathcal{O}(a^{-2})\right],$$

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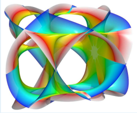
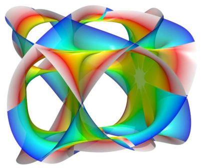

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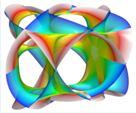
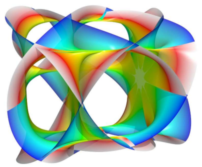
$$\Psi_{HH}(a,\phi) \simeq \begin{cases} \exp\left(\frac{1}{3V(\phi)}\right) \exp\left[\frac{-1}{3V(\phi)}\left(1 - a^2 V(\phi)\right)^{3/2}\right], & a^2 V < 1 \\ \exp\left(\frac{1}{3V(\phi)}\right) \cos\left[\frac{1}{3V(\phi)}\left(a^2 V(\phi) - 1\right)^{3/2} - \frac{\pi}{4}\right], & a^2 V > 1 \end{cases}$$





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- $$\Psi_{HH} = \Psi_- + \Psi_+, \qquad (1)$$
  $$\Psi_\pm \sim e^{\frac{1}{3V(\phi)}} e^{\pm i\left[\frac{1}{3V(\phi)}\left(a^2V(\phi)-1\right)^{3/2} - \frac{\pi}{4}\right]}. \qquad (2)$$

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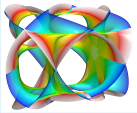

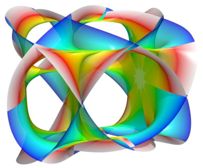

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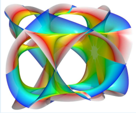
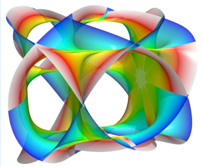

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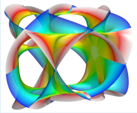
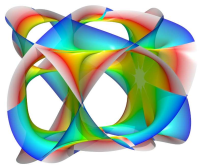
- $$\Psi_V \sim \exp\left[\frac{-i}{3V(\phi)}\left(e^{2\alpha}V(\phi) - 1\right)^{3/2}\right],$$

- wavefunction *complex* in the oscillatory region (the HH is *real*).

Cases and Results

1. Why is QC well motivated for research study?
2. Describe the ‘historical’ path and *who* contributed with *what* within QC
3. How can the Hamiltonian (i.e., the Wheeler–DeWitt) and momentum constraint equations be extracted from a $3 + 1$ description of spacetime?
4. **Can the presence of torsion be of relevance?**
5. What is superspace? What is minisuperspace?
6. Why is so important to discuss the corresponding algebra of constraints? What physical information can be retrieved, namely concerning the geometry of the space time?
7. Indicate the assumptions and properties of the no-boundary and tunneling wave functions for the universe





More Motivation...

more Motif

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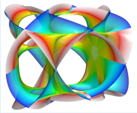
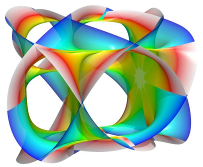

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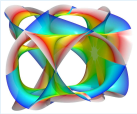
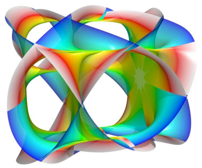

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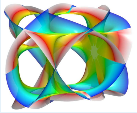
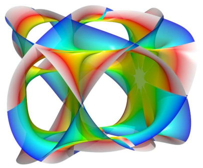
Additional set-up

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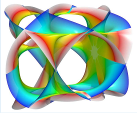
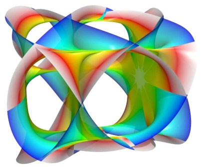

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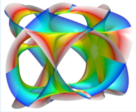
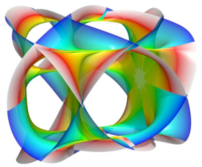

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- $\mathcal{O}(\hbar^0)$
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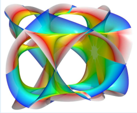
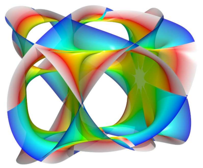

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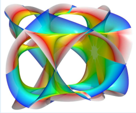
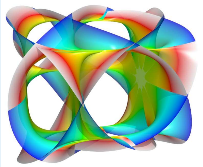

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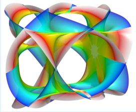 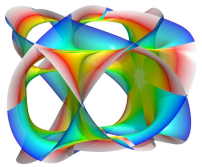

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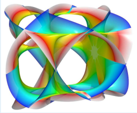

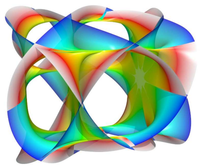

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- S**pacetime correlations and decoherence**:
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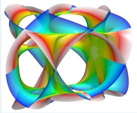 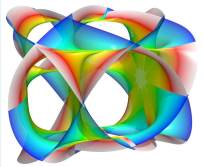

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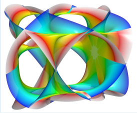

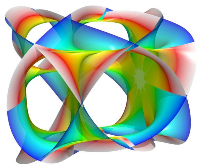

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- Loss of quantum-coherence or decoherence
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$$F_{W(n)}(a, \pi_a) = \int_{-\infty}^{+\infty} \Delta [S'_{(n)}(a_1) S'_{(n)}(a_2)]^{-\frac{1}{2}} e^{-2i\pi_a \Delta} e^{iM_P^2 [S_{(n)}(a_1) - S_{(n)}(a_2)]} \mathcal{I}$$

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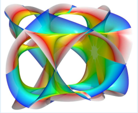
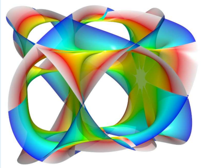

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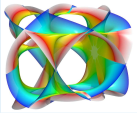
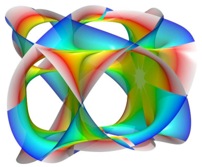

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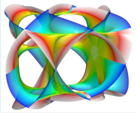
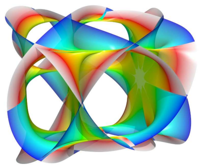

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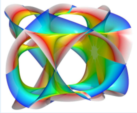
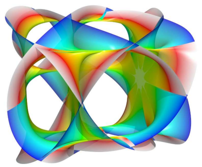

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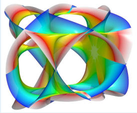
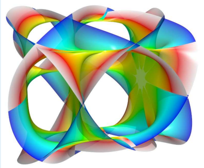

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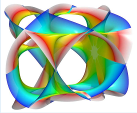 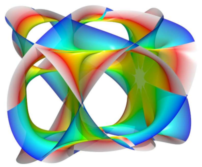

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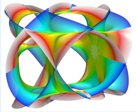 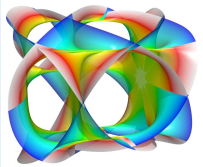

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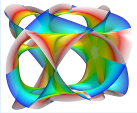
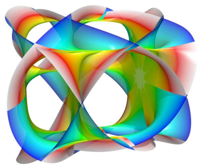

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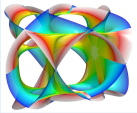
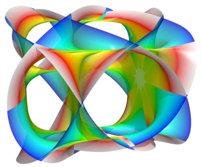

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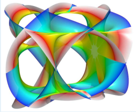
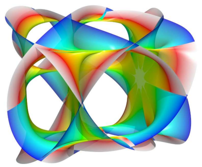

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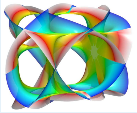
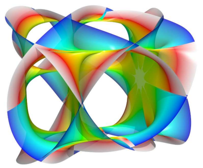

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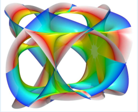
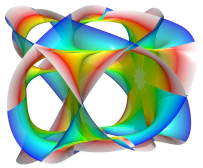

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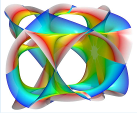
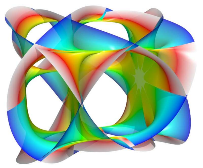

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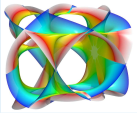
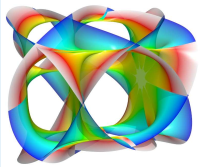
- *In other cases it may not be as simple: WKB components can interfere (e.g., at a turning point of a recollapsing universe); Classical* time *may not be possible to define the issue of back reaction of non-gravitational fields on the Hamilton–Jacobi equation is of pertinence.*

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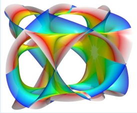
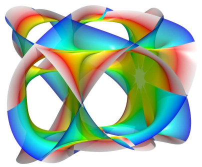

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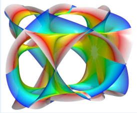
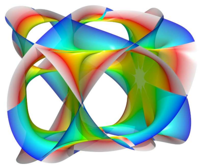

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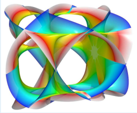
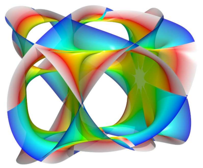

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- $\rightsquigarrow$ reduced density matrix to be diagonal,

again...Settings

- In models containing one single “classical” degree of freedom, the Hamilton–Jacobi equation has only two solutions, generating the same trajectory in opposite directions:
- $$\Psi[\mathcal{O},\mathcal{E}] = \sum_{(n)} C_{(n)}[\mathcal{O}]e^{iM_P^2 S_{(n)}[\mathcal{O}]}\psi_{(n)}[\mathcal{O},\mathcal{E}];$$
- (n) labels the WKB branches (taking only two values, say ± 1, uniquely identifying the two possible solutions of the Hamilton-Jacobi equation)
- Proper decoherence:
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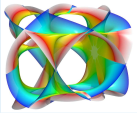
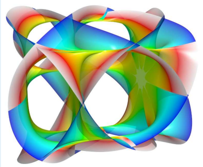
- $\rightsquigarrow$ the different WKB components have negligible interference among the diagonal terms.

again...Settings

- $$\Psi_{(n)}[a,\phi] = e^{iM_P^2 S_{(n)}(a)} C_{(n)}(a)\psi_{(n)}(a,\phi)\ .$$

  $$\rho_R = \sum_{n,n'} e^{iM_P^2[S_{(n)}(a_1)-S_{(n')}(a_2)]} C_{(n)}(a_1) C_{(n')}(a_2) \mathcal{I}_{n,n'}(a_2,a_1)\ ,$$

  where

  $$\mathcal{I}_{n,n'}(a_2,a_1) \equiv \int \tilde{\Psi}^*_{(n')}(a_2,\phi)\tilde{\Psi}_{(n)}(a_1,\phi)d[\phi],$$

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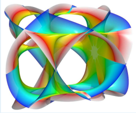

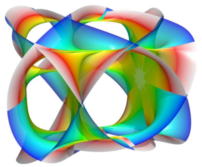

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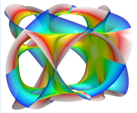
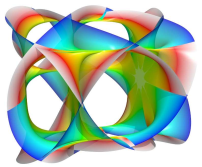
- The term $\mathcal{I}_{n,n'}(a_2,a_1)$ is sometimes designated *Decoherence Functional* and describes the influence of the environment on the system.

again...Settings

- **(small)** inhomogeneous perturbations:

$$h_{ij}(x,t) \equiv a^2(t)\left(\Omega_{ij} + \varepsilon_{ij}\right), \tag{1}$$

$$\phi(x,t) \equiv \phi_0(t) + \delta\phi(x,t) \equiv \phi_0(t) + \frac{1}{\varsigma}\Sigma_{nml} f_{nlm}(t) Q^n_{lm}(x), \tag{2}$$

$$\mathcal{N}(x,t) \equiv \mathcal{N}_0(t) + \delta\mathcal{N}(x,t), \tag{3}$$

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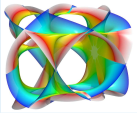
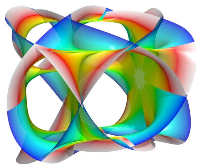

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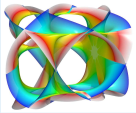
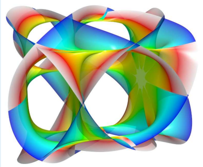

again...Settings

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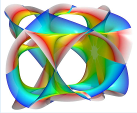
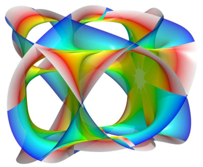

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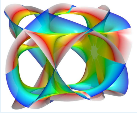 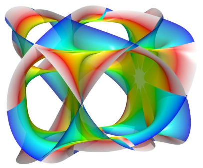

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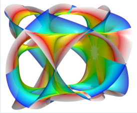
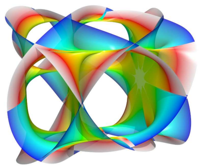

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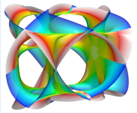
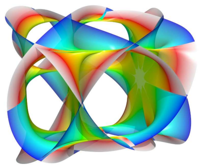

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again...Settings

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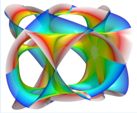

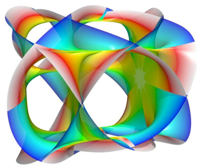

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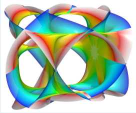
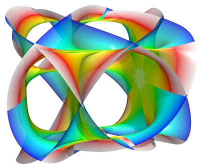

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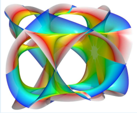
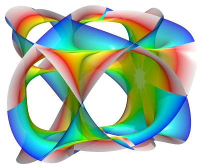

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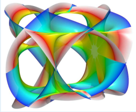
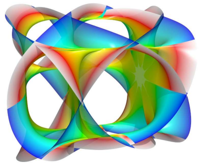

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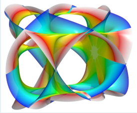
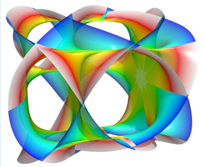

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- *... reduced density matrix; Feynman-Vernon influence functional; Schwinger-Keldish CTP effective action*

again...Settings

- Trajectories in N = 2 minisuperspace: far more complicated;
  A WKB time is now defined as

$$\frac{d}{d\eta} = \mathcal{G}^{XY} \frac{\partial S}{\partial q_X} \frac{\partial}{\partial q_Y} \quad ,$$

with $X, Y = 1, 2$, $q_1 = a, q_2 = \phi$ and $\mathcal{G}^{XY} = \mathrm{diag}(-1, 1)$.

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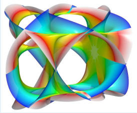
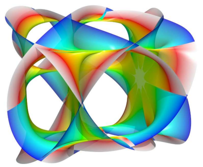
- This implies that the influence functional is actually a functional of two 'histories'; A state $\tilde{\Psi}_{(n)}$ can be interpreted, not as being simply a function of a point in minisuperspace, but instead as a function of the whole history, which corresponds to the only trajectory that belongs to the (n)-WKB branch and goes through that particular point (a, ϕ).

again...Settings

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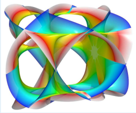 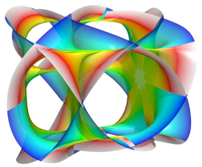

again...Settings

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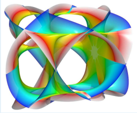
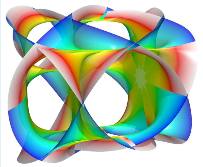
- $\rightsquigarrow a$ and ϕ and their canonical momenta analysed by examining peaks in the *reduced* (!) Wigner function $F_{W1}(q^X, \boldsymbol{\pi}_{q_1}) = \int d\boldsymbol{\pi}_{q_2} F_{W(n)}(q^X; \boldsymbol{\pi}_{q_X})$

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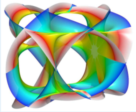
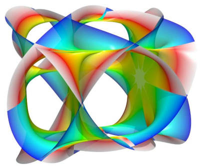
- $\rho[a, \phi; a', \phi'] \simeq$ $\Psi_0 \Psi_0^* \exp\left(-\frac{\mathrm{N}^3}{6a^2}(a - a')^2 - 3m^2\mathrm{N}(a\phi - a'\phi')^2\right)$

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again...Settings

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- A few points:
  - Universe (is more classic as the universe is larger...
  - A classical a leads to a classical ϕ, i.e., interferences within ϕ are negligible if $a \sim a'$;

Of Interest

Cases again...

- De Sitter space and quantum gravity corrections:

Cases again...

- De Sitter space and quantum gravity corrections:
- Thus,
  - conformal transformation 3-metric as $h_{ij} = h^{1/3}\tilde{h}_{ij}$,
  - The Hamilton–Jacobi equation becomes

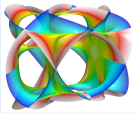
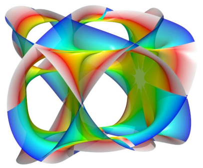
$$-\frac{3\sqrt{h}}{16}\left(\frac{\delta S_0}{\delta\sqrt{h}}\right)^2 + \frac{\tilde{h}_{ik}\tilde{h}_{jl}}{2\sqrt{h}}\frac{\delta S_0}{\delta\tilde{h}_{ij}}\frac{\delta S_0}{\delta\tilde{h}_{kl}} - 2\sqrt{h}({}^{(3)}R - 2\Lambda) = 0.$$

  - ${}^{(3)}R = 0$; $S_0 = S_0(\sqrt{h})$;

$$S_0 = \pm 8\sqrt{\frac{\Lambda}{3}}\int\sqrt{h}d^3x \equiv \pm 8H_0\int\sqrt{h}d^3x,$$

  - “Conservation law”: (functional) Schrödinger equation is

$$i\dot{\Theta} = \int d^3x\left(-\frac{1}{2a^3}\frac{\delta^2}{\delta\phi^2} + \frac{a}{2}(\nabla\phi)^2 + \frac{a^3}{2}m^2\phi^2\right)\Theta.$$

Cases again...

- and...
  - Gaussian ansatz $\Theta = N(t)\exp\left(-\frac{1}{2}\int d\tilde{k}\check{\Omega}(\mathbf{k},t)\check{\chi}_k\check{\chi}_{-k}\right)$;
  - $$y'' + 2\frac{a'}{a}y' + (m^2a^2 + k^2)y = 0,$$
  - At the next order, corrections to the Schrödinger equation:

$$i\hbar\frac{\delta\Theta}{\delta\tau} = \mathcal{H}^m_\perp\Theta - \frac{2\pi G}{\sqrt{h}\Lambda}(\mathcal{H}^m_\perp)^2\Theta - i\hbar\frac{2\pi G}{\Lambda}\frac{\delta}{\delta\tau}\left(\frac{\mathcal{H}^m_\perp}{\sqrt{h}}\right)\Theta.$$

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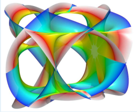
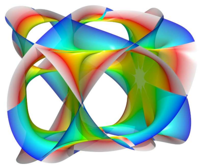
- $\rightsquigarrow$ source of non-unitarity;

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- $\leadsto$ shift

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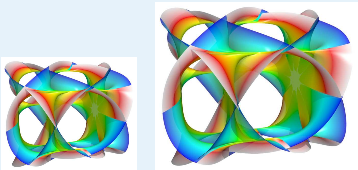

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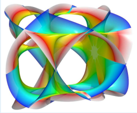
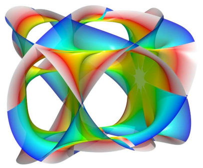

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- ... reducing to the Minkowski vacuum at early times, i. e. $\check{\Omega}$ tends to $\sqrt{n^2+m^2}$ as $t \to -\infty$,

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Cases again...

- **Inflation**:
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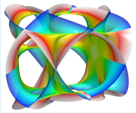 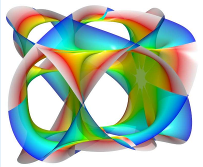

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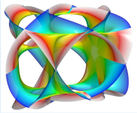
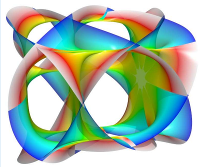

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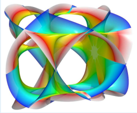
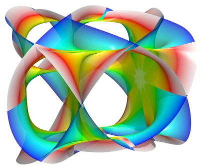
- **Inflation**:
- Can inflation be generic?
- $\leadsto$ *Both WKB limits for the 'no-boundary' and 'tunneling' wavefunctions predict an inflationary stage for a FRW universe*
- The wave functions in the oscillatory region are strongly peaked about the set of classical solutions: $a(t) \sim e^{\sqrt{V}\,t}$, $\phi(t) \simeq \phi_0$

Cases again...

- Making predictions: probability measure must be defined.





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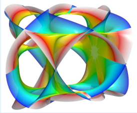
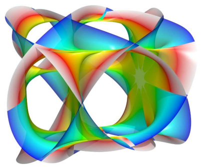

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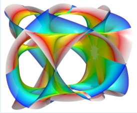
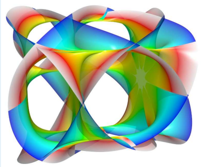

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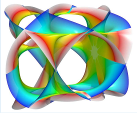
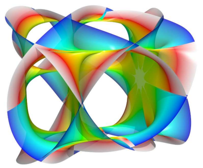
- However:
  - The inner product constructed from J is not positive-definite: *Negative* probabilities become possible;
  - No well-defined definition of positive frequencies in full superspace;
  - Some wavefunctions (e.g., the no-boundary) are real and give $J = 0$.

Cases again...

- Hence, some options we can consider:
  1. Bring Ψ into an operator representation, create and annihilate universes;
  2. Invoke SQC (!), since the constraints of SUGRA are as the Dirac square root of the constraints of general relativity, in the form of first-order equations (However, even so, the problem is not solved:
  3. Use $|\Psi|^2$ directly as a probability measure: For homogeneous minisuperspaces, it is a promising option since quantum cosmology is quantum mechanics with time reparametrization; However, there are examples where the wavefunction is not normalisable;
  4. Consider Ψ associated instead with *conditional* probabilities of eventually finding Ψ in a region A of minisuperspace given that Ψ is found also in another region B.

Cases again...

- WKB semi-classical limit: quite instructive

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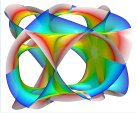
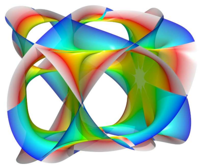

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- $\rightsquigarrow$ each Ψ_n has a conserved current

$$\mathsf{J}_n \simeq |\mathcal{A}_n|^2 \nabla S_n \Leftrightarrow \nabla_X \mathsf{J}_n^X = 0,$$

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  being a conserved probability measure
- For a set of trajectories near the classical trajectory the probability density is positive-definite.

Cases again...

- integrating $d\mathsf{P} = \mathsf{J}_n^X d\Sigma_X$ on the surface separating the tunneling and oscillatory regions (where about $a^2V(\phi) = 1$).

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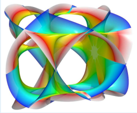
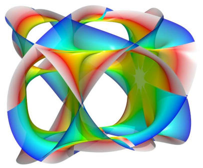

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$$P\left(\phi_0 > \phi_{\rm suf} \mid \phi_1 < \phi_0 < \phi_2\right) = \frac{\int_{\phi_{\rm suff}}^{\phi_2} d\phi_0 \exp\left(\frac{\pm 2}{3V(\phi_0)}\right)}{\int_{\phi_1}^{\phi_2} d\phi_0 \exp\left(\frac{\pm 2}{3V(\phi_0)}\right)},$$

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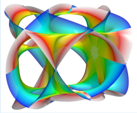
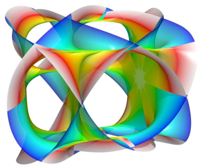
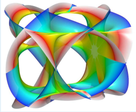
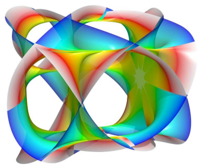
- and
  - 'no-boundary': a probability $P \simeq 1$, obtained;
  - parameters employed, values go beyond the Planck scale, approximation will no longer apply...;
  - Calculations are *model-dependent*, *both* Ψ_{HH} and Ψ_V lead $P \sim 1$...

Topics to revise

Outlook again...

- So,
  1. What are the semiclassical elements extractable?
  2. How can we describe quantum gravitational corrections into a semiclassical background?
  3. Can QC become ‘observational’?
  4. What is ’decoherence’?
  5. What is the relevance of the Bunch-Davies vacuum?
  6. Can inflation and the primordial seeds for structure formation be (satisfactorily) predicted from a wave function of the universe (whatever or with just *some* boundary conditions)?

N=2 SUSY

QC SUSY part-I

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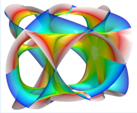
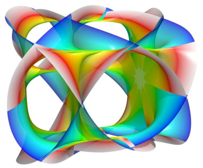
 

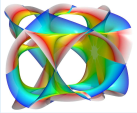
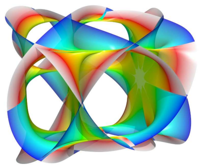
QC SUSY part-I

- Starting from
- $\rightsquigarrow$ bosonic configurations
- $\rightsquigarrow$ General Relativity **or** bosonic sector (strings)
- Setting:

$$ds^2 = -\mathcal{N}^2(t)dt^2 + h_{ij}(t)\omega^i\omega^j$$

$$h_{ij}(t) = \frac{1}{6\pi}e^{2\alpha(t)}\left[e^{2\beta}(t)\right]_{ij} = \frac{1}{6\pi}e^{2\beta^i}\delta_{ij},$$

$$\beta_{ij}(t) \equiv \mathrm{diag}(\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+)$$

QC SUSY part-I

- Hamiltonian

$$
\begin{aligned}
\mathcal{H} &= \frac{1}{2}(-p_\alpha^2 + p_+^2 + p_-^2) + U^{(0)}(\alpha, \beta_+, \beta_-) \\
&= \frac{3}{2}\left[(p_1)^2 + (p_2)^2 + (p_3)^2 - 2p_1p_2 - 2p_1p_3 - 2p_2p_3\right] \\
&+ U^{(0)}(\beta^1, \beta^2, \beta^3), \qquad (1)
\end{aligned}
$$

QC SUSY part-I

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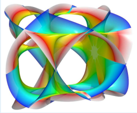
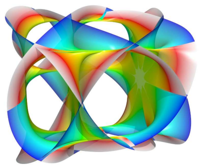
$$\begin{aligned}
\mathcal{H} &= \frac{1}{2}(-p_\alpha^2 + p_+^2 + p_-^2) + U^{(0)}(\alpha, \beta_+, \beta_-) \\
&= \frac{3}{2}\left[(p_1)^2 + (p_2)^2 + (p_3)^2 - 2p_1p_2 - 2p_1p_3 - 2p_2p_3\right] \\
&+ U^{(0)}(\beta^1, \beta^2, \beta^3), \qquad (1)
\end{aligned}$$

- $U^{(0)} \equiv -12\pi^2\, h^{(3)}R$

$$\text{type I}: \qquad U_I^{(0)} \equiv 0 \qquad (2)$$

$$\text{type II}: \qquad U_{II}^{(0)} \equiv \frac{1}{6}e^{4\alpha}e^{-8\beta_+} = \frac{1}{6}e^{4\beta^3}, \qquad (3)$$

$$\begin{aligned}
\text{type IX}: \qquad U_{IX}^{(0)} &\equiv \frac{1}{6}e^{4\alpha}\Big[2e^{4\beta_+}(\cosh(4\sqrt{3}\beta_-) - 1) + e^{-8\beta_+} \\
&\quad -4e^{-2\beta_+}\cosh(2\sqrt{3}\beta_-)\Big] \\
&= \frac{1}{6}\Big[e^{4\beta^1} + e^{4\beta^2} + e^{4\beta^3} - 2e^{2\beta^1+2\beta^2}
\end{aligned}$$

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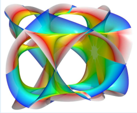 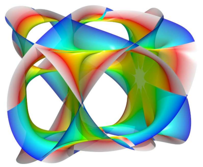

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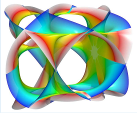

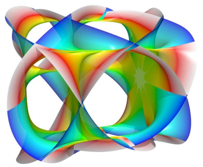

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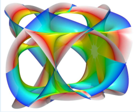
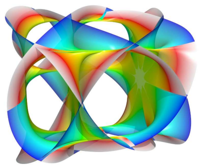



QC SUSY part-I

- Superpotentials W
  - <u>Bianchi type I:</u>

$$\mathrm{W}_I \equiv 0;$$

  - <u>Bianchi type II:</u>

$$\mathrm{W}_{II} \equiv \frac{1}{6} e^{2\alpha - 4\beta_+} = \frac{1}{6} e^{2\beta^3};$$

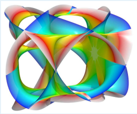
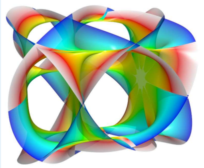
  - <u>Bianchi type IX:</u>

$$\begin{aligned} \mathrm{W}_{IX}^{(0)} &\equiv \frac{1}{6} e^{2\alpha} \left[2e^{2\beta_+} \cosh 2\sqrt{3}\beta_- + e^{-4\beta_+} \right] \\ &= \frac{1}{6} \left(e^{2\beta^1} + e^{2\beta^2} + e^{2\beta^3} \right); \end{aligned} \quad (1)$$

QC SUSY part-I

- Classical Hamiltonian

$$\mathcal{H}_c = \frac{1}{2}\mathcal{G}^{XY}(q)\left(p_X p_Y + \frac{\partial \mathrm{W}}{\partial q^X}\frac{\partial \mathrm{W}}{\partial q^Y}\right).$$

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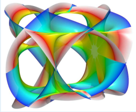
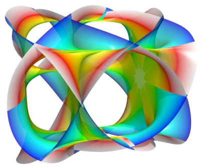

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$$2\mathcal{H} = \tilde{\mathcal{S}}\mathcal{S} + \mathcal{S}\tilde{\mathcal{S}}$$

- where $\mathcal{S}$, $\tilde{\mathcal{S}}$ are linear operators satisfying

$$\mathcal{S}^2 = 0 = \tilde{\mathcal{S}}^2.$$

QC SUSY part-I

- The operators $\mathcal{S}$ and $\tilde{\mathcal{S}}$ have the explicit form

$$\mathcal{S} \equiv \psi^x {e_x}^Y(q)\left(\pi_Y + i\frac{\partial \mathsf{W}}{\partial q^Y}\right) \tag{1}$$

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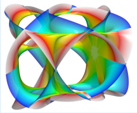
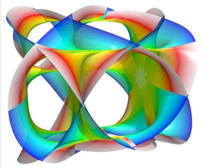

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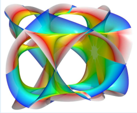
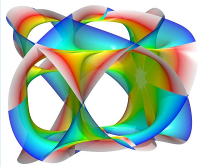

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- They satisfy ${e_x}^Y(q) {e_y}^X(q) \eta^{xy} \equiv \mathcal{G}^{XY}(q)$,

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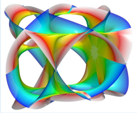

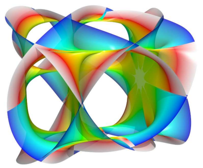

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- satisfying

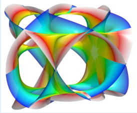
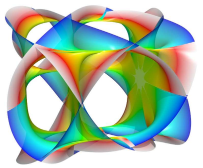
$$\{\psi^x, \psi^y\} = 0 = \{\bar{\psi}_x, \bar{\psi}_y\} \tag{1}$$

$$\{\psi^x, \bar{\psi}_y\} = \delta^x_y. \tag{2}$$

QC SUSY part-I

- Hamiltonian

$$\mathcal{H} = -\frac{\hbar^2}{2}\mathcal{G}^{XY}\frac{\partial}{\partial q^Y}\frac{\partial}{\partial q^X} + \frac{1}{2}\mathcal{G}^{YX}\frac{\partial \mathsf{W}}{\partial q^Y}\frac{\partial \mathsf{W}}{\partial q^X} + \frac{\hbar}{2}e_x{}^Y e_y{}^X \frac{\partial^2 \mathsf{W}}{\partial q^Y \partial q^X}[\bar{\psi}^x, \psi^y] \quad ;$$

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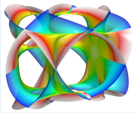
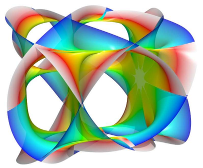

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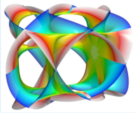
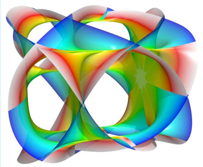

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-

$$[\mathcal{H}, \mathcal{F}] = 0, [\mathcal{S}, \mathcal{F}] = \mathcal{S}, [\tilde{\mathcal{S}}, \mathcal{F}] = -\tilde{\mathcal{S}}.$$

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- In more detail...

$$\mathcal{S}_0 = i\left\{\psi^0\left(-\hbar\frac{\partial}{\partial\alpha} + \frac{\partial \mathrm{W}}{\partial\alpha}\right) + \psi^1\left(-\hbar\frac{\partial}{\partial\beta_+} + \frac{\partial \mathrm{W}}{\partial\beta_+}\right) + \psi^2\left(-\hbar\frac{\partial}{\partial\beta_-} + \frac{\partial \mathrm{W}}{\partial\beta_-}\right)\right\}$$

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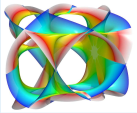 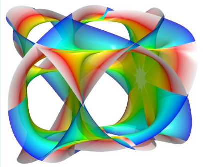

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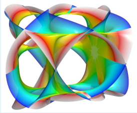
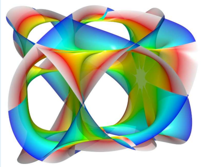

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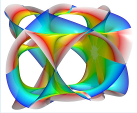
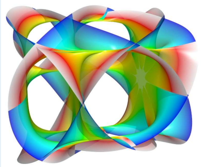
$$\mathsf{A}_\pm = \mathrm{q}_\pm \, e^{\mp \mathsf{W}}, \; \mathsf{B}_X \equiv \frac{\partial \mathsf{f}_+}{\partial \mathrm{q}^{\mathrm{X}}} \, \mathrm{e}^{-\mathsf{W}}, \; \mathsf{C}^Y \equiv \mathcal{G}^{YX} \frac{\partial \mathsf{f}_-}{\partial q^X} \, e^{\mathsf{W}},$$

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QC SUSY part-I

- **Scalar-tensor gravity theories**

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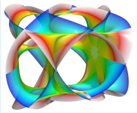
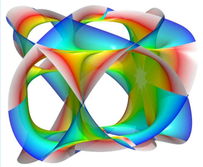

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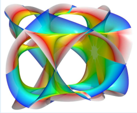

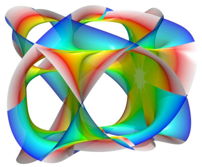

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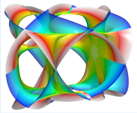
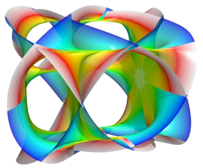

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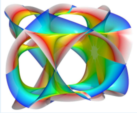
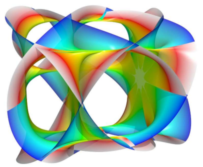

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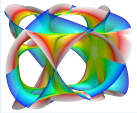
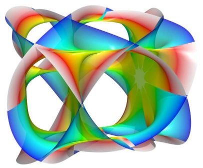

QC SUSY part-I

- Define

$$x \equiv \exp\left[\left(\frac{D-1}{2}+\frac{\gamma}{2}\right)\left(\alpha+\frac{1}{D-2}\left(\frac{1}{\gamma}-1\right)\phi\right)\right],$$
$$y \equiv \exp\left[\left(\frac{D-1}{2}-\frac{\gamma}{2}\right)\left(\alpha-\frac{1}{D-2}\left(\frac{1}{\gamma}+1\right)\phi\right)\right], \qquad (1)$$

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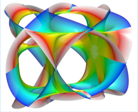
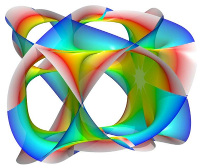

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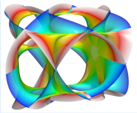
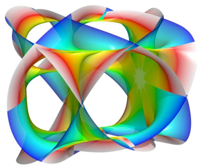
- A second coordinate pair

$$w \equiv \epsilon^{1/2}\left[\frac{D-1+(D-2)\omega}{D+(D-1)\omega}\right]^{1/2}(x-y)$$
$$z \equiv \epsilon^{1/2}\left[\frac{D-1+(D-2)\omega}{D+(D-1)\omega}\right]^{1/2}(x+y), \qquad (2)$$





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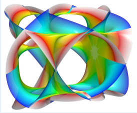
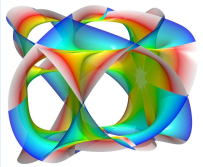

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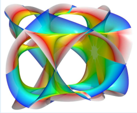

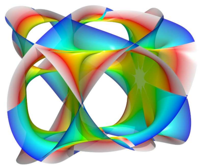

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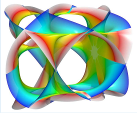
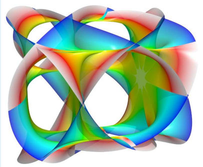

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- Potential is given by

$$U = -\Upsilon(w^2 - z^2)/2.$$

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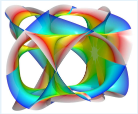
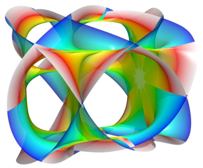
$$\left[\frac{\partial^2}{\partial w^2} - \frac{\partial^2}{\partial z^2} - p(w^2 - z^2)\right]\Psi = 0$$





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- Solutions

$$\Psi_n = H_n(\Upsilon^{1/4} w) H_n(\Upsilon^{1/4} z) e^{-\sqrt{\Upsilon}(w^2+z^2)/2},$$

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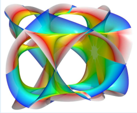
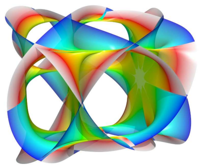

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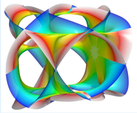
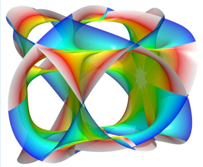

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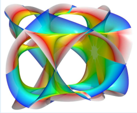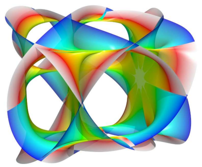

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- $\mathsf{A}_\pm$ empty and filled fermion sectors

SQC and SUGRA

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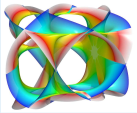

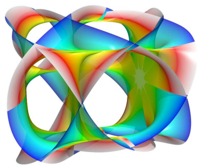

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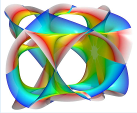
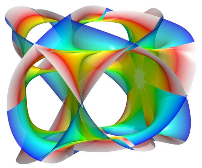

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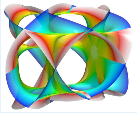
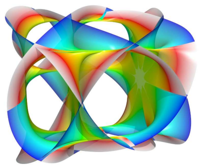

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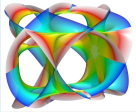

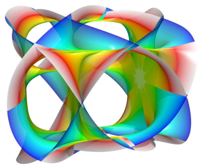

SQC SUGRA

- Important “section”...
- ... reduction of 4-dimensional canonical quantum $N = 1$ SUGRA
- E.g., **FRW models:**
- $\rightsquigarrow$ simplest
- $\rightsquigarrow$ ‘bottom-up’
- *(prudent) approach on (*i*)* how *SQC research should proceeded, (*ii*) methods that are to be applied and (*iii*) the results retrieved: general? Useful?*
- FRW Minisuperspace Reduction *Ansätse*: Consistent with
- $\rightsquigarrow$ supersymmetry,
- $\rightsquigarrow$ Lorentz
- $\rightsquigarrow$ general coordinate transformations,
- Purpose: minisuperspace that will inherit invariance under local time translations, supersymmetry and Lorentz transformation.

SQC SUGRA

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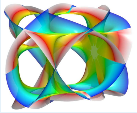
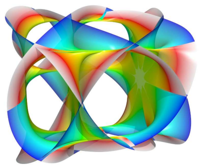

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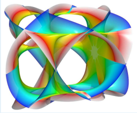
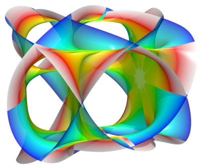

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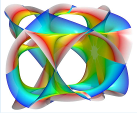

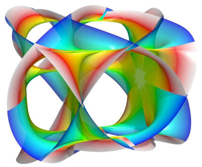

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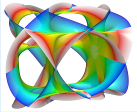
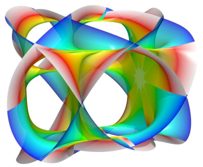

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- Furthermore

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SQC SUGRA

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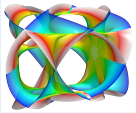
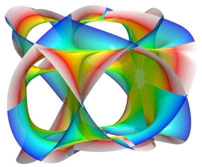

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-

$$
\begin{aligned}
L_{free} \quad = \quad & -\frac{3a}{\mathcal{N}}\left[\dot{a} - \frac{i}{4}\left(\bar{\psi}_{0A'}\bar{\psi}^{A'} - \psi^{A}\psi_{0A}\right)\right]^2 \\
& -i\frac{3}{2}a^3 n_{AA'}\psi^{A}\bar{\psi}^{A} - i\frac{3}{2}a^3 n_{AA'}\bar{\psi}^{A}\dot{\psi}^{A} + 3\mathcal{N}a \\
& +\frac{3}{2}a^2\left(\bar{\psi}_{0A'}\bar{\psi}^{A'} + \psi^{A}\psi_{0A}\right) - \frac{3}{2}\mathcal{N}a^2 n^{AA'}\psi_{A}\bar{\psi}_{A'} \\
& +\frac{3}{16}a^3 n^{AA'}\left(\bar{\psi}_{0A'}\psi_{A}\bar{\psi}^{B'}\bar{\psi}_{B'} + \psi_{0A}\bar{\psi}_{A'}\psi^{B}\psi_{B}\right) \qquad (2)
\end{aligned}
$$

SQC SUGRA

- Hamiltonian

$$H = \mathcal{N}\mathcal{H} + \psi_0^A \mathcal{S}_A + \bar{\mathcal{S}}_{A'}\bar{\psi}_0^{A'} + \mathcal{M}^{AB}\mathcal{J}_{AB} + \bar{\mathcal{M}}^{A'B'}\bar{\mathcal{J}}_{A'B'},$$

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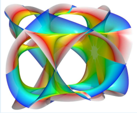
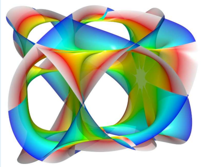

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- Full set of constraints takes a rather *simple* form

$$\mathcal{S}_A = \psi_A \pi_a - 6ia\psi_a, \quad (2)$$

$$\mathcal{H} = -a^{-1}(\pi_a^2 + 36a^2), \quad (3)$$

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- The presence of the free parameters $\rho_A, \bar{\rho}_{A'}$ shows that this

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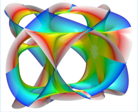 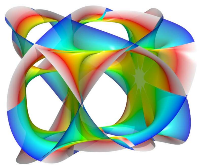

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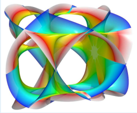
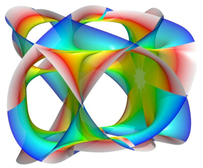

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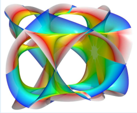
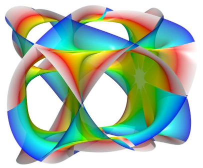

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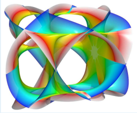
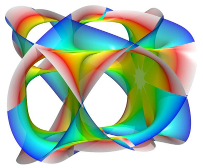

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- I.e., we get a Hartle–Hawking solution for $\mathsf{B} = 0$

Can it be real?!

Semiclassical Expansion

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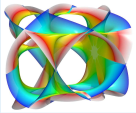
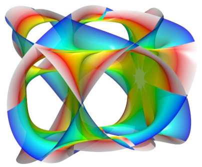

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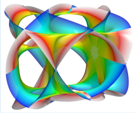 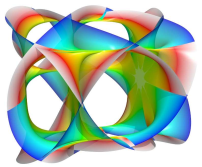

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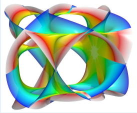
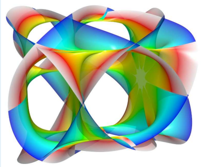

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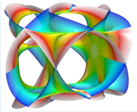
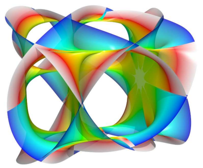
- *The lowest order, G^{-2}, implies (as in the bosonic case) that S_0 is independent on the matter field ϕ. In more formal terms, $S_0 \equiv S_0[e,\psi]$.*

Semiclassical Expansion

- At order G^{-1}

Semiclassical Expansion

- At order G^{-1}
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- $\rightsquigarrow$ Hamilton–Jacobi equation
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$$\frac{1}{2}\mathcal{G}_{ijkl}\frac{\delta S_0^{\mathrm{bos}}}{\delta h_{ij}}\frac{\delta S_0^{\mathrm{bos}}}{\delta h_{kl}}+64\pi n^{AA'}\partial_i e_{AA'j}\frac{\delta S_0^{\mathrm{bos}}}{\delta h_{ij}}+U^{\mathrm{bos}}=0\,.$$

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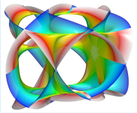
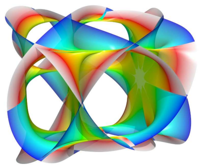
- ... $S_0[e,\psi] = B_0[e] + F_0[e,\psi]$

Semiclassical Expansion

- **The DeWitt *Super* metric**

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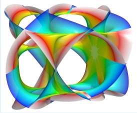
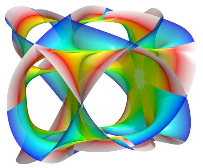

Semiclassical Expansion

- Hamilton–Jacobi equation in *condensed* form

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- Operator:

$$\mathcal{A} \equiv \frac{i}{\sqrt{h}}\epsilon^{ijk} e_i^{BC'} e^{A}{}_{C'l}\left({}^{3\mathrm{s}}\mathcal{D}_j \psi_{Ak}\right)\frac{\delta}{\delta \psi_l^B} \ .$$

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- Dealing with $SuperRiem(\Sigma)$.
- SUSY Hamilton – Jacobi equation induces a spacetime background with both tetrad (graviton) and fermionic (gravitino) terms; “Metric” being $g = g_B + g_S$

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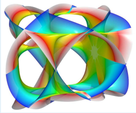
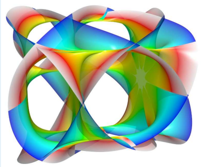
- To order G^0: The Schrödinger Equation

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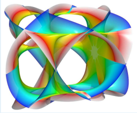
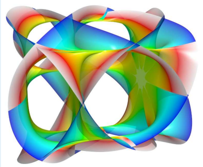

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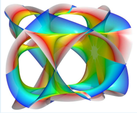
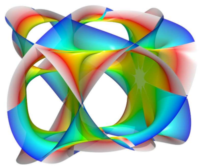

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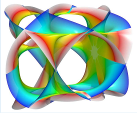 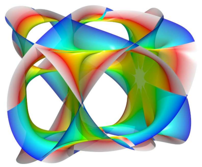

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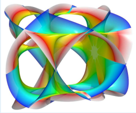
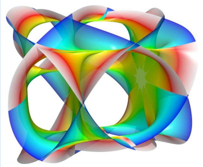
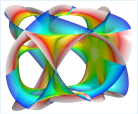
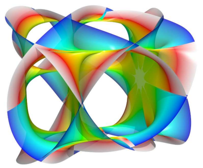



For Research...

Outlook yet again...

- So,
  1. How can the ‘hidden’ $N = 2$ SUSY be retrieved?
  2. And what about when (super)matter includes a scalar field or a Yang–Mills sector?
  3. *How* precisely can we implement a FRW SQC (minisuperspace) from $N = 1$ SUGRA?
  4. *How* can scalar (super)matter be included?
  5. Why is the case of Bianchi models with a cosmological constant still an issue?
  6. How does the DeWitt supermetric relates to the usual DeWitt metric?
  7. What are the main features of SUGRA (quantum) corrections into the Schrödinger equation?
  8. Can SQC become ‘observational’?