Superradiant Instability of AdS black holes

Akihiro Ishibashi

VIII Black Holes Workshop

TECNICO, LISBON, 21 Dec. 2015

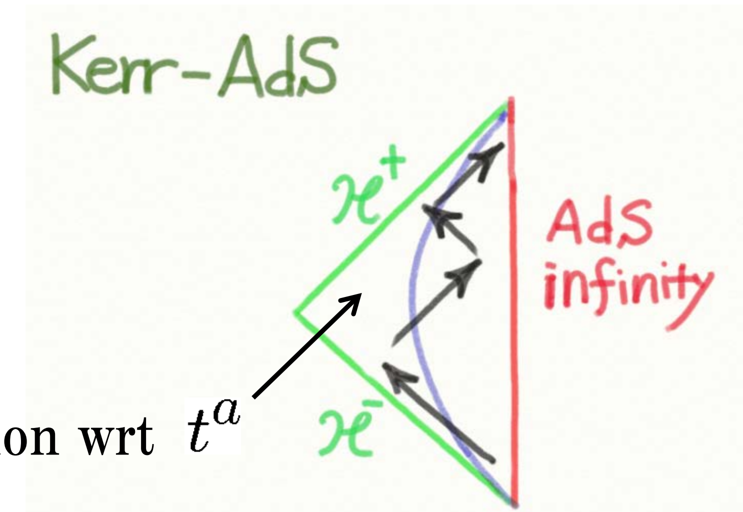
w/ S.R. Green, S. Hollands, R.M. Wald

arXiv: 1512.02644

$$d \geq 4$$

Instability of rotating AdS black holes

- Rotating AdS BH $\Rightarrow$ Superradiant instability

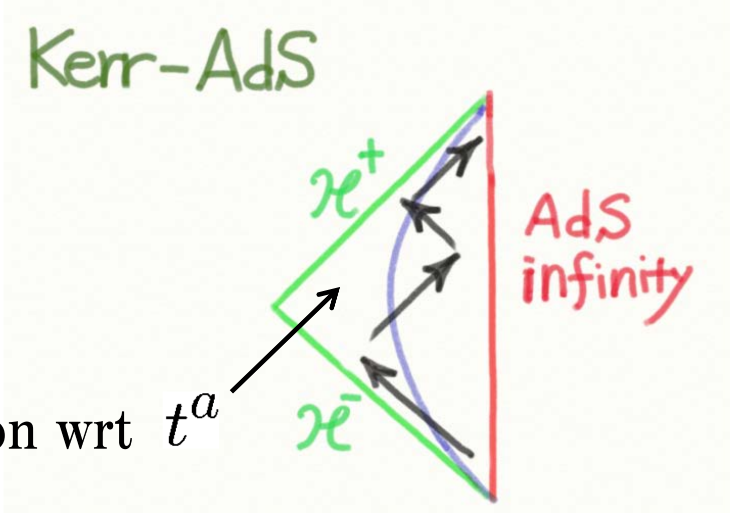


Ergoregion wrt t^a

Hawking-Reall 99, Cardoso-Dias 04
Cardoso-Dias-Yoshida 06,
Kodama 07 Murata-Soda 08
Kunduri-Lucietti-Reall 06
Uchikata-Yoshida-Futamase 09
Kodama-Konoplya-Zhidenko 09
Cardoso-Dias-Hartnett-Lehner-Santos 14
… etc.
See e.g review Brito-Cardoso-Pani 15

Instability of rotating AdS black holes

- Rotating AdS BH $\Rightarrow$ Superradiant instability



Ergoregion wrt t^a

Hawking-Reall 99, Cardoso-Dias 04
Cardoso-Dias-Yoshida 06,
Kodama 07 Murata-Soda 08
Kunduri-Lucietti-Reall 06
Uchikata-Yoshida-Futamase 09
Kodama-Konoplya-Zhidenko 09
Cardoso-Dias-Hartnett-Lehner-Santos 14
… etc.
See e.g review Brito-Cardoso-Pani 15

- Hawking-Reall bound

Slow-rotation $\Omega_H \le 1/\ell$ ➡ Horizon Killing vector field K^a
$\Rightarrow$ causal everywhere outside the horizon

➡ $E = -\int dS^a K^b T_{ab} \ge 0$ Stable wrt scalar field

Def. An ergoregion of asymptotically AdS black hole

A region where the horizon Killing vector field is spacelike

Slow-rotation → No ergoregion wrt $K^a = t^a + \Omega^I \phi_I^a$

Fast-rotation $\Omega_H > 1/\ell$ there exists an ergoregion near AdS infinity

Def. An ergoregion of asymptotically AdS black hole

A region where the horizon Killing vector field is spacelike

Slow-rotation → No ergoregion wrt $K^a = t^a + \Omega^I \phi_I^a$

Fast-rotation $\Omega_H > 1/\ell$ there exists an ergoregion near AdS infinity

Theorem Green-Hollands-AI-Wald

Any asymptotically globally AdS black hole with Killing horizon is unstable if it admits an ergoregion with respect to the horizon Killing field K^a

Sketch of proof.

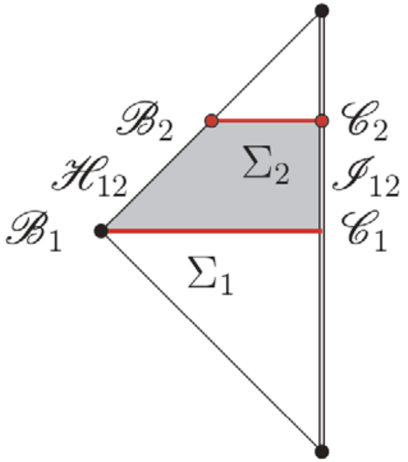
Symplectic form for gravitational perturbations

$$W(\Sigma;\gamma_1,\gamma_2)\equiv\int_\Sigma \star w(g;\gamma_1,\gamma_2)$$

$$w^a=\frac{1}{16\pi}P^{abcdef}\left(\gamma_{2\,bc}\nabla_d\gamma_{1\,ef}-\gamma_{1\,bc}\nabla_d\gamma_{2\,ef}\right)$$

The Canonical energy of the initial data for perturbations

$$\mathcal{E}_K(\gamma)=W_\Sigma(g;\gamma,\pounds_K\gamma)$$



Sketch of proof.

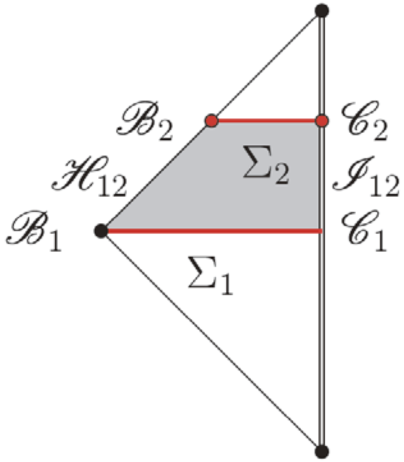
Symplectic form for gravitational perturbations

$$W(\Sigma;\gamma_1,\gamma_2) \equiv \int_\Sigma \star w(g;\gamma_1,\gamma_2)$$

$$w^a = \frac{1}{16\pi} P^{abcdef}\left(\gamma_{2\,bc}\nabla_d \gamma_{1\,ef} - \gamma_{1\,bc}\nabla_d \gamma_{2\,ef}\right)$$

The Canonical energy of the initial data for perturbations

$$\mathcal{E}_K(\gamma) = W_\Sigma(g;\gamma,\pounds_K \gamma)$$

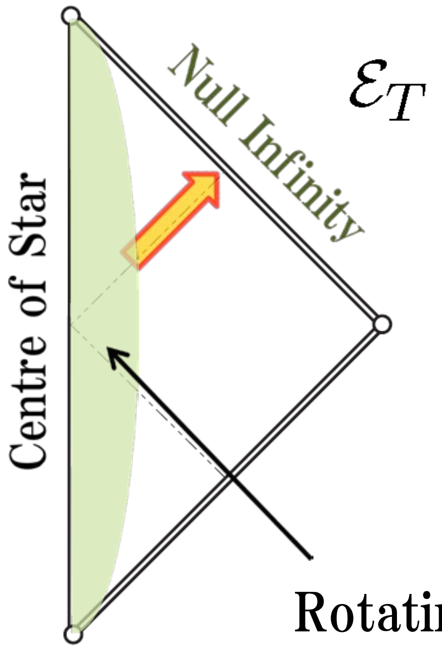


The canonical energy has the following properties:

1. Gauge-invariant
2. Monotonically decreasing $\mathcal{E}_{\Sigma_2} \leq \mathcal{E}_{\Sigma_1}$ if the flux at boundary is positive

c.f. Canonical energy and stability analysis

Positive Flux at Null Infinity
wrt Stationary Killing field t^a

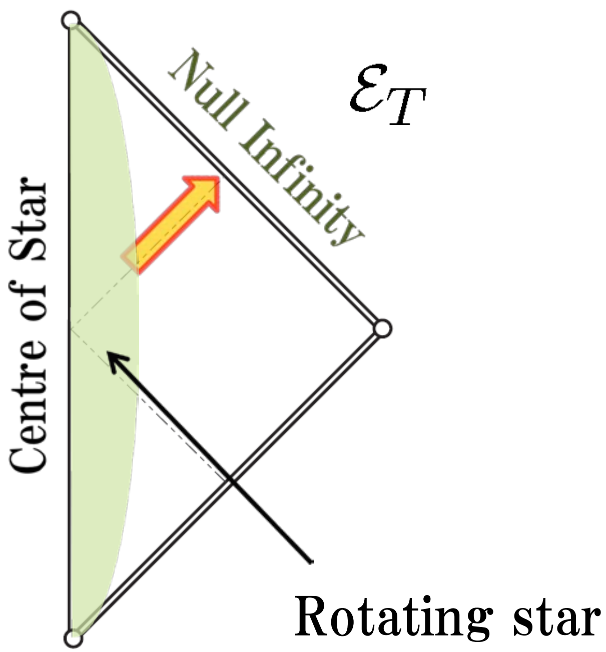


Instability of
rotating relativistic stars

Friedman 78

c.f. Canonical energy and stability analysis

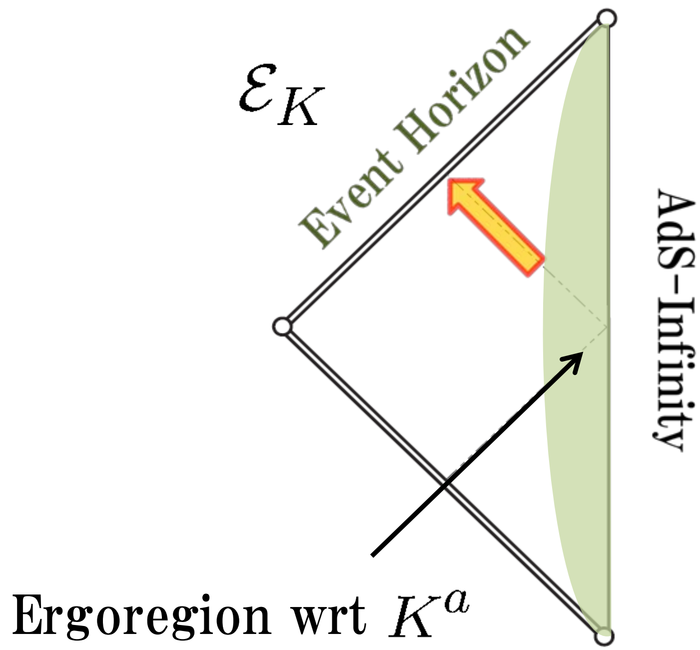
Positive Flux at Null Infinity
wrt Stationary Killing field t^a



Instability of
rotating relativistic stars

Friedman 78

Positive Flux at Event horizon
wrt Horizon Killing field K^a



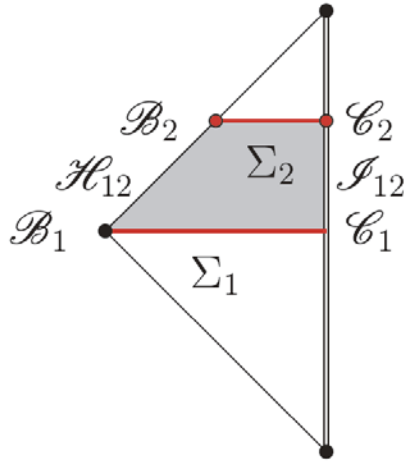
Superradiant instability of
AdS black holes

Sketch of proof.

Symplectic form for gravitational perturbations

$$W(\Sigma;\gamma_1,\gamma_2)\equiv\int_\Sigma \star w(g;\gamma_1,\gamma_2)$$

$$w^a=\frac{1}{16\pi}P^{abcdef}(\gamma_{2\,bc}\nabla_d\gamma_{1\,ef}-\gamma_{1\,bc}\nabla_d\gamma_{2\,ef})$$



The Canonical energy of the initial data for perturbations

$$\mathcal{E}_K(\gamma)=W_\Sigma(g;\gamma,\pounds_K\gamma)$$

The canonical energy has the following properties:

1. Gauge-invariant
2. Monotonically decreasing $\mathcal{E}_{\Sigma_2}\leq\mathcal{E}_{\Sigma_1}$ if the flux at boundary is positive

- wish to show the existence of γ_{ab} with which $\mathcal{E}_K(\gamma)<0$ in ergoregion

WKB method ⇒ expansion with respect to $1/\omega \ll 1$

$$\gamma_{ab} = \exp(i\omega\chi)\left[\gamma_{ab}^{(0)} + \frac{1}{\omega}\gamma_{ab}^{(1)} + \ldots\right]$$

Eikonal equation： $\nabla^a\chi\nabla_a\chi = 0$

In ergoregion： $K^a\nabla_a\chi > 0$

WKB method ⇒ expansion with respect to $1/\omega \ll 1$

$$\gamma_{ab} = \exp(i\omega\chi)\left[\gamma_{ab}^{(0)} + \frac{1}{\omega}\gamma_{ab}^{(1)} + \dots\right]$$

Eikonal equation : $\nabla^a\chi\nabla_a\chi = 0$

In ergoregion : $K^a\nabla_a\chi > 0$

The canonical energy can take the form in the ergoregion

$$\mathcal{E} = -\frac{\omega^2}{8\pi}\int (K^a\nabla_a\chi)\cdot\|\gamma\|^2\cdot\sin^2(\omega\chi) + O(\omega)$$

WKB method ⇒ expansion with respect to $1/\omega \ll 1$

$$\gamma_{ab} = \exp(i\omega\chi)\left[\gamma_{ab}^{(0)} + \frac{1}{\omega}\gamma_{ab}^{(1)} + \ldots\right]$$

Eikonal equation : $\nabla^a\chi\nabla_a\chi = 0$

In ergoregion : $K^a\nabla_a\chi > 0$

The canonical energy can take the form in the ergoregion

$$\mathcal{E} = -\frac{\omega^2}{8\pi}\int (K^a\nabla_a\chi)\cdot\|\gamma\|^2\cdot\sin^2(\omega\chi) + O(\omega)$$

$\mathcal{E}_K < 0$ As large negative as one wants
in ergoregion, where $K^a\nabla_a\chi > 0$

WKB method ⇒ expansion with respect to $1/\omega \ll 1$

$$\gamma_{ab} = \exp(i\omega\chi)\left[\gamma_{ab}^{(0)} + \frac{1}{\omega}\gamma_{ab}^{(1)} + \dots\right]$$

Eikonal equation： $\nabla^a \chi \nabla_a \chi = 0$

In ergoregion： $K^a \nabla_a \chi > 0$

The canonical energy can take the form in the ergoregion

$$\mathcal{E} = -\frac{\omega^2}{8\pi}\int (K^a \nabla_a \chi) \cdot \|\gamma\|^2 \cdot \sin^2(\omega\chi) + O(\omega)$$

WKB solution does not satisfy the linearized constraints, but the failure is as small as one wants.
By applying the Corvino-Schoen method, we can correct the initial data for the perturbations so that it satisfies the constraints. □

Summary

We have shown that any asymptotically globally AdS black hole with an ergoregion is unstable.

Summary

We have shown that any asymptotically globally AdS black hole with an ergoregion is unstable.

We require only that the black hole have a single Killing vector field normal to the horizon and no restrictions on gravitational perturbations (not necessary to be axisymmetric as in the asymptotically flat case).

Summary

We have shown that any asymptotically globally AdS black hole with an ergoregion is unstable.

We require only that the black hole have a single Killing vector field normal to the horizon and no restrictions on gravitational perturbations (not necessary to be axisymmetric as in the asymptotically flat case).

It is implausible to consider that the perturbaiton would approach a time periodic solution.