Absorption and superradiance of the massive and charged scalar field by Reissner-Nordström black holes

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Contents

Introduction

- Matzner, 1968: First work on scattering by black holes.
- Bekenstein, 1973: “Extraction of energy and charge from a black hole”.

Reissner-Nordström spacetime

The Reissner-Nordström line element is given by

$$ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \qquad (1)$$

where

$$f(r) = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right). \qquad (2)$$

Klein-Gordon equation

The Klein-Gordon equation is

$$(\nabla_\nu - iqA_\nu)(\nabla^\nu - iqA^\nu)\Phi = \mu^2\Phi. \quad (3)$$

The solution to this equation, in the Reissner-Nordström spacetime is

$$\Phi_{\omega l} = \frac{\psi_{\omega l}(r)}{r} P_l(\theta) e^{-i\omega t}, \quad \text{for } \omega > \mu, \quad (4)$$

where $P_l(\theta)$ is a legendre polynomial and $\psi_{\omega l}(r)$ obeys the radial equation

$$f\frac{d}{dr}\left(f\frac{d}{dr}\psi_{\omega l}\right) + \left[\omega^2 - V(r)\right]\psi_{\omega l} = 0, \quad (5)$$

with

$$V(r) = \left[\left(\omega - \frac{qQ}{r}\right)^2 - f\left(\mu^2 + \frac{l(l+1)}{r^2} + \frac{f'}{r}\right)\right] \quad (6)$$

We can now rewrite Eq. (5) using the tortoise coordinate, defined as

$$\frac{d}{dr^*} = f\frac{d}{dr}. \tag{7}$$

We find then, for the radial equation,

$$\frac{d^2}{dr_*^2}\psi_{\omega l} + \left[\omega^2 - V(r)\right]\psi_{\omega l} = 0. \tag{8}$$

We find the following solutions for the asymptotic limits:

$$\psi_{\omega l}(r) \approx \begin{cases} T_{\omega l}\, e^{-i\xi r_*}, & \text{for } r \to r_+, \\ e^{-i\rho r_*} + R_{\omega l} e^{i\rho r_*}, & \text{for } r \to \infty, \end{cases} \tag{9}$$

where $\xi \equiv \omega - qQ/r_+$ and $\rho \equiv \sqrt{\omega^2 - \mu^2}$. The coefficients in these solutions can be identified as the reflection and transmission coefficients, which obey

$$|R_{\omega l}|^2 + \frac{\xi}{\rho}|T_{\omega l}|^2 = 1. \tag{10}$$

The absorption cross section is given by

$$\sigma = \sum_{l=0}^{\infty} \sigma_l = \sum_{l=0}^{\infty} \frac{\pi}{\rho^2}(2l+1)(1-|R_{\omega l}|^2) \qquad (11)$$

High-frequency limit

According to the eikonal approximation we can write

$$\sigma_{hf} = \pi b_c^2, \tag{12}$$

where b_c is the critical impact parameter. Using the geodesics equations we find

$$\left(\frac{du}{d\phi}\right)^2 = -f(u)u^2 + (1 - f(u))\frac{\mu^2}{L^2} + \frac{Q^2u^2q^2}{L^2} - \frac{2QqEu}{L^2} + \frac{E^2 - \mu^2}{L^2}, \tag{13}$$

where $u \equiv 1/r$ and $f(u) = 1 - 2Mu + Q^2u^2$. E and L are the energy and the angular momentum of the particle, respectively, which, in the semiclassical limit can be associated to $E \to \omega$ and $L \to l + 1/2$, respectively.

Low-frequency limit

To solve the Klein-Gordon equation in this limit we consider three different regions:

- Region *I*: for $r \approx r_+$;
- Region *II*: for $\omega \approx m \approx 0$;
- Region *III*: for $r \to \infty$.

We then make an interpolation between the regions and find the low-frequency absorption cross section, given by

$$\sigma_{lf} = \frac{\mathcal{A}}{\rho}\left(\omega - \frac{qQ}{r_+}\right), \tag{14}$$

where $\mathcal{A} = 4\pi r_+^2$ is the area of the black hole.

Numerical results

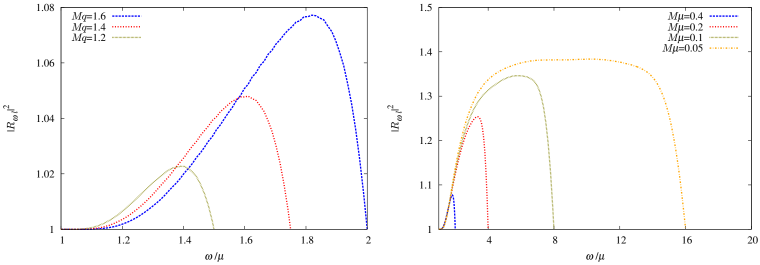

Figure : Reflection coefficient as a function of the frequency for $Q/M = 0.8$ and $l = 0$. For the left plot we fix $M\mu = 0.4$ and for the right plot $Mq = 1.6$.

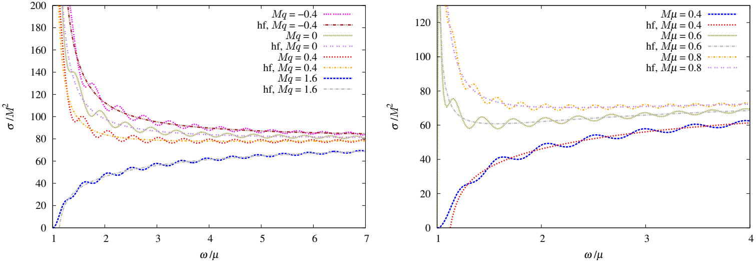

Figure : Total absorption cross section as a function of the frequency, for $Q/M = 0.4$. For the left panel we choose $M\mu = 0.4$. For the right panel we choose $Mq = 1.6$.

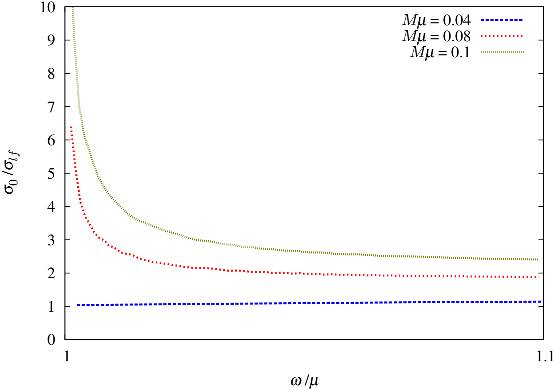

Figure : Comparison between the partial absorption cross section for $l = 0$ (σ_0) obtained numerically, and the low-frequency approximation (σ_{lf}) for different choices of $M\mu$. We have chosen $Q/M = 0.4$ and $Mq = 0.1$

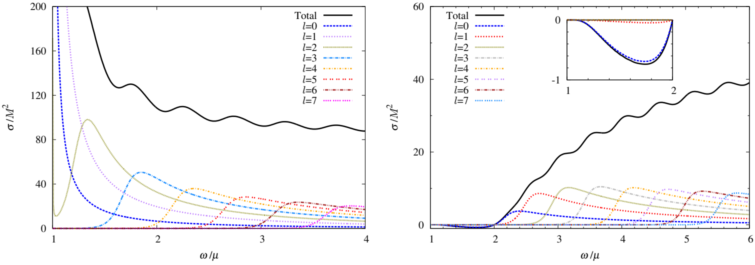

Figure : Partial and total absorption cross sections for $M\mu = 0.4$. For the left plot $Mq = -0.4$ and $Q/M = 0.4$, while for the right plot $Mq = 1.6$ and $Q/M = 0.8$.

Conclusion

- The total absorption cross section oscillates around the geometric-optics result.
- As we increase the charge coupling qQ, the absorption cross section gets smaller. This is due to the presence of a repulsive electromagnetic interaction (the Lorentz force) for $qQ > 0$ competing with the gravitational interaction, causing the decrease of the absorption.
- The Lorentz repulsion force can render finite the low-frequency limit of the absorption cross section.
- The result for the low-frequency limit can be regarded as a generalization of the one obtained for the chargeless massive scalar field.
- Planar scalar waves can be superradiantly amplified in this case.