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Gravity's Rainbow and the Tolman-Oppenheimer-Volkoff Equation

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Plan of the Talk
(Work in Progress!!!)

- Motivation and Introduction to Gravity’s Rainbow
- Gravity’s Rainbow and TOV
- Two simple solutions for the isotropic case (Dev-Gleiser energy density potential)
- Possible origin of the Dev-Gleiser potential
- Conclusions and Outlooks

Ordinary TOV

$$\frac{dp_r}{dr} = -\left(\rho + \frac{p_r}{c^2}\right)\frac{4\pi G r^3 p_r / c^2 + Gm(r)}{r^2\left[1 - 2Gm(r)/rc^2\right]} + \frac{2}{r}\left(p_t - p_r\right)$$

$$\frac{dm}{dr} = 4\pi\rho(r)r^2 \qquad \frac{dp_r}{dr} + \left(\rho(r)c^2 + p_r\right)\Phi'(r) = 0$$

Two exact solutions for the isotropic case

$$\rho(r) = \text{constant} \qquad \rho(r) = \frac{3c^2}{56\pi G r^2} \quad \text{Misner-Zapolsky}$$

Why do we need to modify gravity at very high energies???

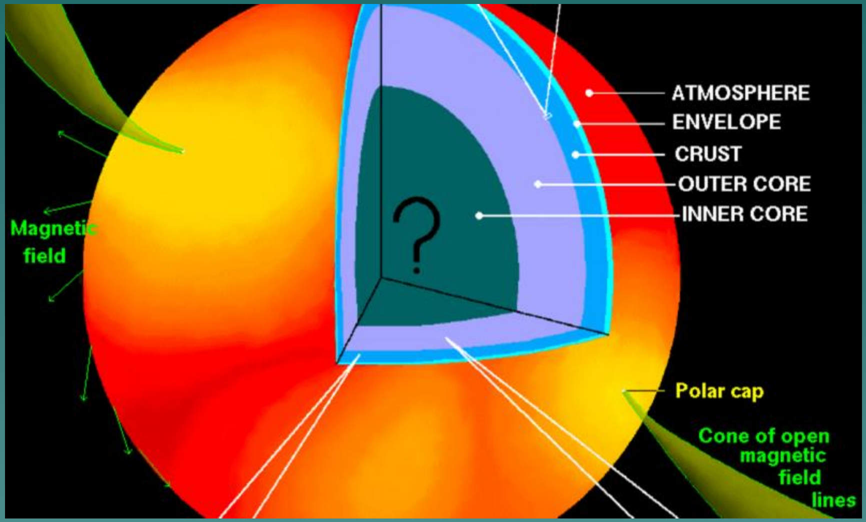
The Core of a Compact Star

Examples of Stars with QG Modifications

GUP Quantum Corrections
P. Wang, H.Yang and X. Zhang
PLB 718 (2012), 265

Asymptotically Safe Gravity
See F.Saueressig

Planck Stars
C.Rovelli and F.Vidotto
IJMPD 23 (2014) 12, 1442026



Credit: Dany P Page

Gravity's Rainbow

Gravity's Rainbow

Doubly Special Relativity

G. Amelino-Camelia, Int.J.Mod.Phys. D 11, 35 (2002); gr-qc/001205.
G. Amelino-Camelia, Phys.Lett. B 510, 255 (2001); hep-th/0012238.

$$E^2 g_1^2\left(E/E_P\right) - p^2 g_2^2\left(E/E_P\right) = m^2 \qquad \lim_{E/E_P\to 0} g_1\left(E/E_P\right) = \lim_{E/E_P\to 0} g_2\left(E/E_P\right) = 1$$

Curved Space Proposal → *Gravity's Rainbow*
[J. Magueijo and L. Smolin, Class. Quant. Grav. 21, 1725 (2004) arXiv:gr-qc/0305055].

$$ds^2 = -\frac{\exp\left(2\Phi\left(r\right)\right)dt^2}{g_1^2\left(E/E_P\right)} + \frac{dr^2}{\left(1-\frac{2Gm\left(r\right)}{rc^2}\right)g_2^2\left(E/E_P\right)} + \frac{r^2}{g_2^2\left(E/E_P\right)}d\theta^2 + \frac{r^2}{g_2^2\left(E/E_P\right)}\sin^2\theta d\varphi^2$$

$\Phi\left(r\right)$ is the redshift function $\qquad m\left(r\right)$ is the mass function

TOV+ Gravity's Rainbow

$$\frac{dp_r}{dr} = -\left(\rho + \frac{p_r}{c^2}\right)\frac{4\pi G r^3 p_r / c^2 g_2^2\left(E/E_P\right) + Gm\left(r\right)}{r^2\left[1 - 2Gm\left(r\right)/rc^2\right]}$$

$$\frac{dm}{dr} = \frac{4\pi\rho\left(r\right)r^2}{g_2^2\left(E/E_P\right)} \qquad E \text{ does not depend on } r$$

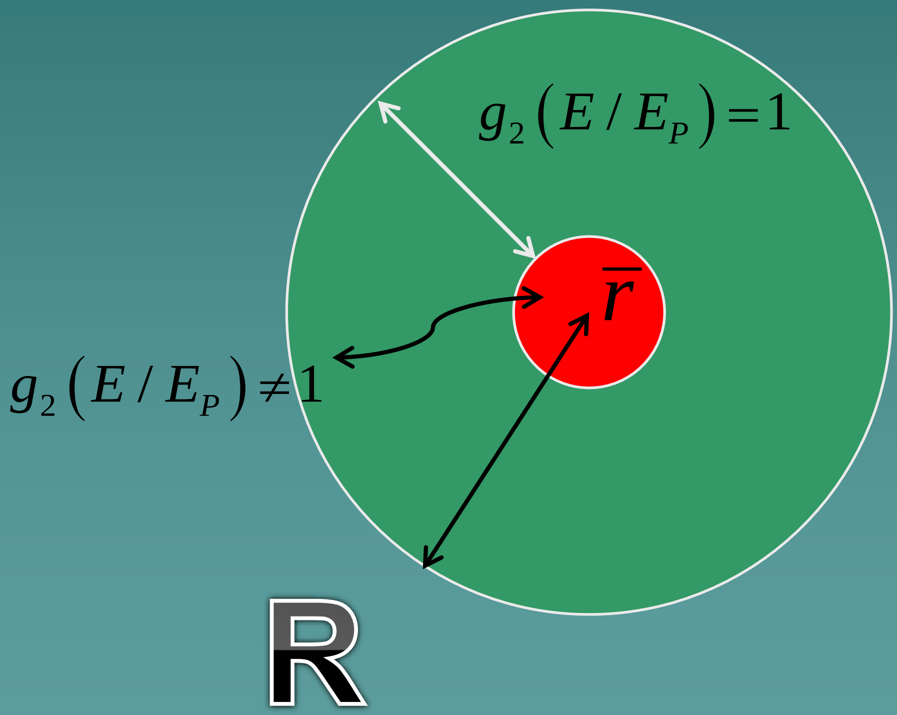
Isotropic Case

$$\frac{dp_r}{dr} + \left(\rho\left(r\right)c^2 + p_r\right)\Phi'\left(r\right) = 0$$

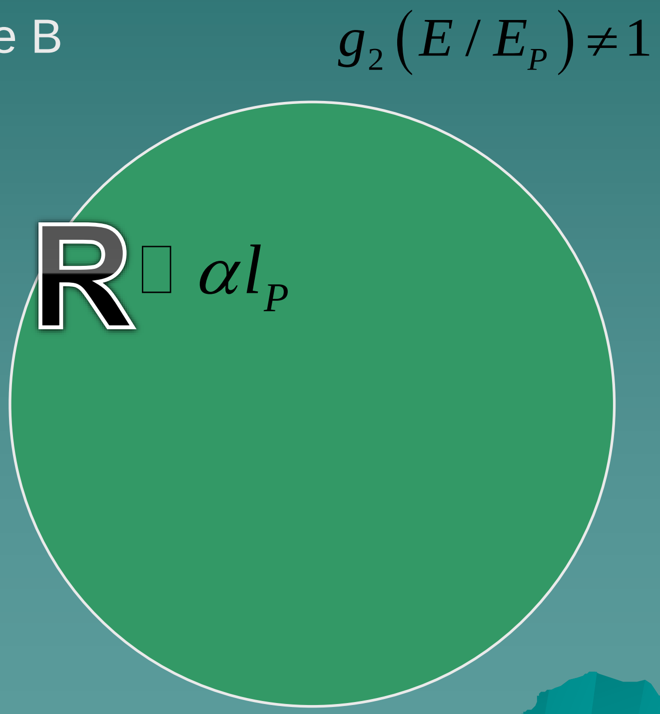
Invariant

Compact Stars in Gravity’s Rainbow

Case A



Case B



$\alpha \propto 10^6$ to fix ideas!!!

TOV+ Gravity's Rainbow

$$\frac{dp_r}{dr} = -\left(\rho + \frac{p_r}{c^2}\right)\frac{4\pi G r^3 p_r / c^2 g_2^2\left(E/E_P\right) + Gm\left(r\right)}{r^2\left[1 - 2Gm\left(r\right)/rc^2\right]}$$

Isotropic Case

$$\frac{dm}{dr} = \frac{4\pi\rho\left(r\right)r^2}{g_2^2\left(E/E_P\right)}$$

$$\frac{dp_r}{dr} + \left(\rho\left(r\right)c^2 + p_r\right)\Phi'\left(r\right) = 0$$

Invariant

Simple case $\rho\left(r\right) = \text{constant}$

Solving TOV+ Gravity's Rainbow

Simple case $\rho(r) = \text{constant}$

$$p_r(r) = \rho c^2 \frac{\left(\sqrt{3c^2 g_2^2(E/E_{\text{Pl}}) - \kappa\rho r^2} - \sqrt{3c^2 g_2^2(E/E_{\text{Pl}}) - \kappa\rho R^2}\right)}{\left(3\sqrt{3c^2 g_2^2(E/E_{\text{Pl}}) - \kappa\rho R^2} - \sqrt{3c^2 g_2^2(E/E_{\text{Pl}}) - \kappa\rho r^2}\right)}.$$

$\kappa = 8\pi G$

P(R)=0

$$M = \frac{4\pi\rho}{3g_2^2(E/E_{\text{Pl}})}R^3 \qquad \Longrightarrow \qquad \rho = g_2^2(E/E_{\text{Pl}})\frac{3M}{4\pi R^3} = g_2^2(E/E_{\text{Pl}})\tilde{\rho},$$

$\rho(r)$ mass density in ordinary GR

Solving TOV+ Gravity's Rainbow

$$p_r(r) = g_2^2(E/E_{\rm Pl})\tilde{\rho}c^2 \frac{\left(\sqrt{3c^2-\kappa\tilde{\rho}r^2}-\sqrt{3c^2-\kappa\tilde{\rho}R^2}\right)}{\left(3\sqrt{3c^2-\kappa\tilde{\rho}R^2}-\sqrt{3c^2-\kappa\tilde{\rho}r^2}\right)}$$

$$= g_2^2\left(E/E_P\right)\tilde{p}_r\left(r\right) \qquad p_r\left(r\right) \quad \text{pressure in ordinary GR}$$

Or in terms of M and R

$$p_r(r) = \frac{3M g_2^2(E/E_{\rm Pl})}{4\pi R^3} c^2 \frac{\sqrt{c^2-2MGr^2/R^3}-\sqrt{c^2-2MG/R}}{3\sqrt{c^2-2MG/R}-\sqrt{c^2-2MGr^2/R^3}}$$

$$= g_2^2\left(E/E_P\right)\tilde{p}_r\left(r\right)$$

Solving TOV+ Gravity's Rainbow

Buchdahl-Bondi bound is preserved

$$M < \frac{4}{9}\frac{c^2}{G}R$$

The central pressure becomes

$$p_c \simeq g_2^2(E/E_{\mathrm{Pl}})\frac{2\pi G\tilde{\rho}^2 R^2}{3} -$$

$$R \,\square\, \alpha l_P$$

p_c is large but finite ⇒ $g_2\left(E/E_P\right) \,\square\, 1$

Solving TOV+ Gravity's Rainbow

$\Phi(r)$ is invariant with respect to $g_2(E/E_{\rm Pl})$

Surface Redshift

$$z = \frac{\triangle\lambda}{\lambda} = \frac{g_1(E/E_{\rm Pl})}{\exp[\Phi(R)]} - 1 = \frac{g_1(E/E_{\rm Pl})}{\sqrt{1-\frac{2MG}{Rc^2}}} - 1$$

$$z \leq z_{\rm max} = 3g_1(E/E_{\rm Pl}) - 1,$$

$$z_{\rm max} = 2 + 3\alpha\frac{E}{E_p} + O\left(\frac{E}{E_p}\right)^2$$

$$\alpha = \frac{1}{E_P}\left(\frac{dg_1\left(E/E_P\right)}{dE}\right)_{E=0} \qquad \alpha \gtrless 0$$

TOV+ Gravity's Rainbow

$$\frac{dp_r}{dr} = -\left(\rho + \frac{p_r}{c^2}\right)\frac{4\pi G r^3 p_r / c^2 g_2^2\left(E/E_P\right) + Gm\left(r\right)}{r^2\left[1 - 2Gm\left(r\right)/rc^2\right]} \qquad \frac{dm}{dr} = \frac{4\pi\rho\left(r\right)r^2}{g_2^2\left(E/E_P\right)}$$

Variable Energy Density Case

$$\rho\left(r\right) = \frac{3c^2}{56\pi G r^2}$$ Misner-Zapolsky

EoS

$$p_r\left(r\right) = \omega\rho\left(r\right)$$

$$c = 1 \quad g_2\left(E/E_P\right) = 1$$

$$\omega = 1/3 \quad \omega = 3$$

TOV

$$1 = \frac{3\left(c^2 + \omega\right)^2}{4\omega c^2\left[7g_2^2\left(E/E_P\right) - 3\right]}$$

$$\omega \Rightarrow \omega \equiv g_2\left(E/E_P\right)$$

$$m(r) = \frac{3\pi c^2 r}{14G}$$

TOV+ Gravity's Rainbow

$$1=\frac{3\left(c^2+\omega\right)^2}{4\omega c^2\left[7g_2^2\left(E/E_P\right)-3\right]}$$

$$\omega_\pm=c^2\left[\frac{14}{3}g_2^2\left(E/E_P\right)-3\pm\frac{2}{3}\sqrt{49g_2^4\left(E/E_P\right)-63g_2^2\left(E/E_P\right)+18}\right]$$

$$g_2\left(E/E_P\right)\gg 1\Rightarrow\begin{cases}\omega_+\cong\dfrac{28c^2}{3}g_2^2\left(E/E_P\right)\\[2ex]\omega_-\cong\dfrac{3c^2}{28g_2^2\left(E/E_P\right)}\end{cases}$$

$$g_2\left(E/E_P\right)\ll 1$$

$$\omega_\pm=c^2\left[-3\pm2\sqrt{2}\right]$$

Dark Energy Star???

Dust???

Vanishing Mass??

$$M=m(R)=\frac{3\pi c^2R}{g_2^2(E/E_{\rm Pl})14G}.$$

Infinite Mass??

Possible Origin of the Dev-Gleiser Potential using the Wheeler-De Witt Equation

$$\min_{\Psi[h]}\left[\frac{1}{V}\frac{\int D\mu[h]\Psi^*[h]\int_\Sigma d^3x\Lambda_\Sigma\Psi[h]}{\int D\mu[h]\Psi^*[h]\Psi[h]}\right]=-\frac{\Lambda}{\kappa}$$

$$\hat{\Lambda}_\Sigma=(2\kappa)G_{ijkl}\pi^{ij}\pi^{kl}-\frac{\sqrt{g}}{2\kappa}R$$

$$V=\int_\Sigma d^3\vec{x}\sqrt{g}$$

$$D\mu[h]=D\left[h_{ij}^{\perp}\right]D\left[\xi_j^T\right]D[h]J$$

Energy Density

1 Loop graviton contribution

Induced Cosmological "Constant"

Solve this infinite dimensional PDE with a Variational Approach without matter fields contribution

Ψ is a trial wave functional of the gaussian type

Schrödinger Picture

Spectrum of Λ depending on the metric

Energy (Density) Levels

For a Minkowski or a de Sitter background in a low energy limit

$$\frac{\Lambda}{\kappa}\simeq A+\frac{B}{Gr^2}$$

Dev-Gleiser

$$g_1\left(E/E_{\rm Pl}\right)=\left(1+\beta\frac{E}{E_{\rm Pl}}+\delta\frac{E^2}{E_{\rm Pl}^2}+\gamma\frac{E^3}{E_{\rm Pl}^3}\right)\exp\left(-\alpha\frac{E^2}{E_{\rm Pl}^2}\right)\qquad g_2\left(E/E_{\rm Pl}\right)=1.$$

Conclusions and Perspectives

- Gravity's Rainbow modifies TOV.
- Ordinary Compact Stars can turn to Dark Stars or Dust??? Planck Stars??? ← Deeper Investigation!!
- Investigation on the constant energy density term and on the complete Dev-Gleiser Energy density profile.
- Zero Point Energy Calculations in Gravity's Rainbow induce an Energy density profile of the Dev-Gleiser form.
- Including corrections to the Dev-Gleiser Energy density profile.
- Anisotropy and Polytropic relations must be included.