Near-horizon geometry and integral quantities for strongly “magnetised” black holes

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Black holes in magnetic universes

- Harrison transformation applied to the Minkowski spacetime yields magnetic universe of Bonnor and Melvin
- "Magnetised" black holes from asymptotically flat ones; MKN metric

$$\boldsymbol{g} = |\Lambda|^2\,\Sigma\left[-\frac{\Delta}{\mathscr{A}}\,\mathbf{d}t^2 + \frac{\mathbf{d}r^2}{\Delta} + \mathbf{d}\vartheta^2\right] + \frac{\mathscr{A}}{\Sigma\,|\Lambda|^2}\sin^2\vartheta\,(\mathbf{d}\varphi - \omega\,\mathbf{d}t)^2$$

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- Function Λ is constructed from Ernst potentials

$$\Lambda = 1 + \frac{1}{4}B^2\left(\frac{\mathscr{A} + a^2Q^2\left(1+\cos^2\vartheta\right)}{\Sigma}\sin^2\vartheta + Q^2\cos^2\vartheta\right) + \\ + \frac{BQ}{\Sigma}\left[ar\sin^2\vartheta - \mathrm{i}\left(r^2+a^2\right)\cos\vartheta\right] - \\ - \frac{\mathrm{i}}{2}B^2a\cos\vartheta\left[M\left(3-\cos^2\vartheta\right) + \frac{Ma^2\sin^2\vartheta - Q^2r}{\Sigma}\sin^2\vartheta\right]$$

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- Complicated form of dragging potential ω and of electromagnetic field

The potential Φ_0 is given by

$$\Phi_0 = \frac{\Phi_0^{(0)} + \Phi_0^{(1)}B + \Phi_0^{(2)}B^2 + \Phi_0^{(3)}B^3}{4\Sigma}, \tag{B.17}$$

where

$$\begin{aligned}
\Phi_0^{(0)} &= 4[-qr(r^2+a^2) + ap\Delta\cos\theta],\\
\Phi_0^{(1)} &= -6a\tilde{q}^2(r^2+a^2+\Delta\cos^2\theta),\\
\Phi_0^{(2)} &= -3q[(r+2m)a^4 - (r^2+4mr+\Delta\cos^2\theta)r^3 + a^2(2\tilde{q}^2(r+2m) - 6mr^2 - 8m^2r\\
&\quad -3\Delta r\cos^2\theta)] + 3p\Delta[3ar^2 + a^3 + a(a^2+\tilde{q}^2-r^2)\cos^2\theta]\cos\theta,\\
\Phi_0^{(3)} &= -\tfrac{1}{2}a\{4a^4m^2 + a^4\tilde{q}^2 + 12a^2m^2\tilde{q}^2 + 2a^2\tilde{q}^4 + 2a^4mr - 24a^2m^3r + 4a^2m\tilde{q}^2r\\
&\quad -24a^2m^2r^2 - 4a^2mr^3 - 12m^2r^4 - \tilde{q}^2r^4 - 6mr^5 - 6r\Delta[2m(r^2+a^2)\\
&\quad -\tilde{q}^2r]\cos^2\theta + \Delta(\tilde{q}^4 - 3\tilde{q}^2r^2 + 2mr^3 + a^2(4m^2+\tilde{q}^2-6mr)]\cos^4\theta\}.
\end{aligned} \tag{B.18}$$

The potential $\Phi_3 = \chi$ is given by

$$\Phi_3 = \chi = \frac{\chi_{(0)} + \chi_{(1)}B + \chi_{(2)}B^2 + \chi_{(3)}B^3}{R^2H}, \tag{B.15}$$

where

$$\begin{aligned}
\chi_{(0)} &= aqr\sin^2\theta - p(r^2+a^2)\cos\theta,\\
\chi_{(1)} &= \tfrac{1}{2}[\Sigma\sin^2\theta + 3\tilde{q}^2(a^2+r^2\cos^2\theta)],\\
\chi_{(2)} &= \tfrac{3}{4}aqr(r^2+a^2)\sin^4\theta - \tfrac{3}{4}p(r^2+a^2)^2\sin^2\theta\cos\theta + 3a^2pmr\sin^2\theta\cos\theta\\
&\quad +\tfrac{3}{2}aqm[r^2(3-\cos^2\theta)\cos^2\theta + a^2(1+\cos^2\theta)] - \tfrac{3}{4}aq\tilde{q}^2r\sin^2\theta\cos^2\theta\\
&\quad -\tfrac{3}{4}p\tilde{q}^2[(r^2-a^2)\cos^2\theta + 2a^2]\cos\theta,\\
\chi_{(3)} &= \tfrac{1}{8}R^2(r^2+a^2)^2\sin^4\theta + \tfrac{1}{2}a^2mr(r^2+a^2)\sin^6\theta - \tfrac{1}{2}a^2\tilde{q}^2mr(5-\cos^2\theta)\sin^2\theta\cos^2\theta\\
&\quad +\tfrac{1}{2}a^2m^2[r^2(3-\cos^2\theta)^2\cos^2\theta + a^2(1+\cos^2\theta)^2]\\
&\quad +\tfrac{1}{4}\tilde{q}^2(r^2+a^2)[r^2+a^2+a^2\sin^2\theta]\sin^2\theta\cos^2\theta\\
&\quad +\tfrac{1}{8}\tilde{q}^4[r^2\cos^2\theta + a^2(2-\cos^2\theta)^2]\cos^2\theta.
\end{aligned} \tag{B.16}$$

The quantity ω is given by

$$\omega = \frac{(2mr-\tilde{q}^2)a + \omega_{(1)}B + \omega_{(2)}B^2 + \omega_{(3)}B^3 + \omega_{(4)}B^4}{\Sigma}, \tag{B.8}$$

where

$$\begin{aligned}
\omega_{(1)} &= -2qr(r^2+a^2) + 2ap\Delta\cos\theta,\\
\omega_{(2)} &= -\tfrac{3}{2}a\tilde{q}^2(r^2+a^2+\Delta\cos^2\theta),\\
\omega_{(3)} &= 4qm^2a^2r + \tfrac{1}{2}ap\tilde{q}^4\cos^3\theta + \tfrac{1}{2}qr(r^2+a^2)[r^2-a^2+(r^2+3a^2)\cos^2\theta]\\
&\quad +\tfrac{1}{2}ap(r^2+a^2)[3r^2+a^2-(r^2-a^2)\cos^2\theta]\cos\theta\\
&\quad +\tfrac{1}{2}q\tilde{q}^2r[(r^2+3a^2)\cos^2\theta - 2a^2] + \tfrac{1}{2}ap\tilde{q}^2[3r^2+a^2+2a^2\cos^2\theta]\cos\theta\\
&\quad -am\tilde{q}^2(2aq + pr\cos^3\theta) + qm[r^4 - a^4 + r^2(r^2+3a^2)\sin^2\theta]\\
&\quad -apmr[2R^2 + (r^2+a^2)\sin^2\theta]\cos\theta,\\
\omega_{(4)} &= \tfrac{1}{2}a^3m^3r(3+\cos^4\theta) - \tfrac{1}{16}a\tilde{q}^6\cos^4\theta - \tfrac{1}{8}a\tilde{q}^4[r^2(2+\sin^2\theta)\cos^2\theta + a^2(1+\cos^4\theta)]\\
&\quad +\tfrac{1}{16}a\tilde{q}^2(r^2+a^2)[r^2(1-6\cos^2\theta+3\cos^4\theta) - a^2(1+\cos^4\theta)]\\
&\quad -\tfrac{1}{4}a^3m^2\tilde{q}^2(3+\cos^4\theta) + \tfrac{1}{4}am^2[r^4(3-6\cos^2\theta+\cos^4\theta)\\
&\quad +2a^2r^2(3\sin^2\theta - 2\cos^4\theta) - a^4(1+\cos^4\theta)]\\
&\quad +\tfrac{1}{8}am\tilde{q}^4r\cos^4\theta + \tfrac{1}{4}am\tilde{q}^2r[2r^2(3-\cos^2\theta)\cos^2\theta\\
&\quad -a^2(1-3\cos^2\theta-2\cos^4\theta)] + \tfrac{1}{8}amr(r^2+a^2)[r^2(3+6\cos^2\theta-\cos^4\theta)\\
&\quad -a^2(1-6\cos^2\theta-3\cos^4\theta)].
\end{aligned} \tag{B.9}$$

Black holes in magnetic universes II

- **Gibbons**, G. W., **Mujtaba**, A. H., **Pope**, C. N. *Ergoregions in magnetized black hole spacetimes*. Classical and Quantum Gravity, 30. 2013.
- Problem with interpretation of the azimuthal coordinate, axis regular if

$$\varphi \in \left(0, 2\pi\left[1 + \frac{3}{2}B^2Q^2 + 2B^3MQa + B^4\left(\frac{1}{16}Q^4 + M^2a^2\right)\right]\right)$$

- Non-flat asymptotics resembling, but not identical to the Bonnor-Melvin magnetic universe
- Approximately flat region if

$$r_+ \ll r \ll \frac{1}{B}$$

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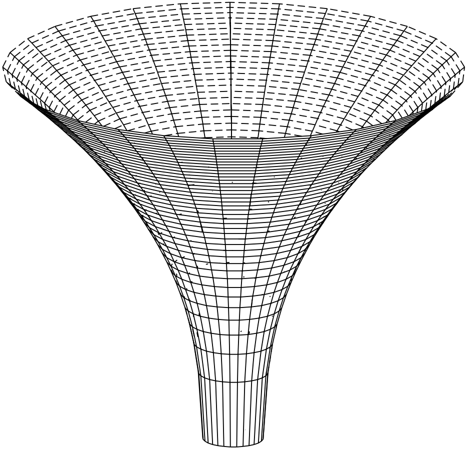
- Note: Generalised electrostatic potential $\phi = -A_t - \omega A_\varphi$

Infinite throat



(Reissner-Nordström, $Q = M, t = \text{const.}$)

Recipe for near-horizon limit of the metric

- **Carter**, Brandon. *Black hole equilibrium states*. Les Houches Lectures, 1972.
- **Bardeen**, J., **Horowitz**, G. T. *Extreme Kerr throat geometry: A vacuum analog of* $\mathrm{AdS}_2 \times \mathscr{S}^2$. Physical Review D, $\underline{60}$. 1999.

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- One can factorise out a function Δ from some metric coefficients of a general axially symmetric stationary metric

$$\boldsymbol{g} = -\Delta \tilde{N}^2 \, \mathbf{d}t^2 + g_{\varphi\varphi} \left(\mathbf{d}\varphi - \omega \, \mathbf{d}t\right)^2 + \frac{\tilde{g}_{rr}}{\Delta} \, \mathbf{d}r^2 + g_{\vartheta\vartheta} \, \mathbf{d}\vartheta^2$$

- Let us assume that the black hole is extremal and coordinate r is chosen so that hypersurface $r = r_0$ is the degenerate horizon
- Then set

$$\Delta = (r - r_0)^2$$

Recipe for near-horizon limit of the metric II

- Coordinate transformation depending on a limiting parameter p

$$r = r_0 + p\chi$$
$$t = \frac{\tau}{p}$$

- Assume that $\tilde{N}, \tilde{g}_{rr}, g_{\varphi\varphi}, g_{\vartheta\vartheta}$ as functions of χ have a finite, nonzero limit for $p \to 0$, so that just this expression is left to resolve

$$\mathbf{d}\varphi - \omega\,\mathbf{d}t$$

- Expansion of the dragging potential

$$\omega \doteq \omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} (r - r_0) = \omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} p\chi$$

Recipe for near-horizon limit of the metric III

- Plugging in the expansion we get

$$\mathbf{d}\varphi - \omega\,\mathbf{d}t \doteq \mathbf{d}\varphi - \left(\omega_{\mathrm{H}} + \left.\frac{\partial\omega}{\partial r}\right|_{r_0} p\chi\right)\frac{\mathbf{d}\tau}{p} = \mathbf{d}\varphi - \frac{\omega_{\mathrm{H}}}{p}\,\mathbf{d}\tau - \left.\frac{\partial\omega}{\partial r}\right|_{r_0}\chi\,\mathbf{d}\tau$$

- It is necessary to add „rewinding“ of the azimuthal angle

$$\varphi = \psi + \frac{\omega_{\mathrm{H}}}{p}\tau$$

Recipe for near-horizon limit of the metric III

- Plugging in the expansion we get

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- It is necessary to add „rewinding“ of the azimuthal angle

$$\varphi = \psi + \frac{\omega_{\mathrm{H}}}{p}\tau$$

- For the Kerr-Newman solution we get the following limiting metric

$$\boldsymbol{g} = \left[Q^2 + a^2\left(1+\cos^2\vartheta\right)\right]\left(-\frac{\chi^2}{\left(Q^2+2a^2\right)^2}\,\mathbf{d}\tau^2 + \mathbf{d}\vartheta^2 + \frac{\mathbf{d}\chi^2}{\chi^2}\right) + \\ + \frac{\left(Q^2+2a^2\right)^2}{Q^2+a^2\left(1+\cos^2\vartheta\right)}\sin^2\vartheta\left(\mathbf{d}\psi + \frac{2a\sqrt{Q^2+a^2}\chi}{\left(Q^2+2a^2\right)^2}\,\mathbf{d}\tau\right)^2$$

Near-horizon limit of the electromagnetic field

- Again, we should expand to the first order and rearrange the terms

$$\boldsymbol{A} = A_t\,\mathbf{d}t + A_\varphi\,\mathbf{d}\varphi \doteq$$

$$\doteq \left(A_t|_{r_0} + \left.\frac{\partial A_t}{\partial r}\right|_{r_0} p\chi\right)\frac{\mathbf{d}\tau}{p} + \left(A_\varphi|_{r_0} + \left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0} p\chi\right)\left(\mathbf{d}\psi + \frac{\omega_{\mathrm{H}}}{p}\,\mathbf{d}\tau\right) \doteq$$

$$\doteq \left(A_t|_{r_0} + \omega_{\mathrm{H}}\,A_\varphi|_{r_0}\right)\frac{\mathbf{d}\tau}{p} + \left(\left.\frac{\partial A_t}{\partial r}\right|_{r_0} + \omega_{\mathrm{H}}\left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0}\right)\chi\,\mathbf{d}\tau + A_\varphi|_{r_0}\,\mathbf{d}\psi =$$

$$= -\frac{\phi_{\mathrm{H}}}{p}\,\mathbf{d}\tau + \left(\left.\frac{\partial A_t}{\partial r}\right|_{r_0} + \omega_{\mathrm{H}}\left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0}\right)\chi\,\mathbf{d}\tau + A_\varphi|_{r_0}\,\mathbf{d}\psi$$

- We assume the generalised electrostatic potential of the horizon ϕ_{H} to be constant, so we can get rid of the singular term by changing gauge

Near-horizon limit of the electromagnetic field

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$$\doteq \left(A_t|_{r_0} + \omega_\mathrm{H}\, A_\varphi|_{r_0} \right) \frac{\mathbf{d}\tau}{p} + \left(\left.\frac{\partial A_t}{\partial r}\right|_{r_0} + \omega_\mathrm{H} \left.\frac{\partial A_\varphi}{\partial r}\right|_{r_0} \right) \chi \,\mathbf{d}\tau + A_\varphi|_{r_0} \,\mathbf{d}\psi =$$

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- We assume the generalised electrostatic potential of the horizon ϕ_H to be constant, so we can get rid of the singular term by changing gauge
- For Kerr-Newman we get

$$\boldsymbol{A} = \frac{Q}{Q^2 + a^2\left(1 + \cos^2\vartheta\right)} \left(\frac{Q^2 + a^2 \sin^2\vartheta}{Q^2 + 2a^2} \chi \,\mathbf{d}\tau + a\sqrt{Q^2 + a^2}\sin^2\vartheta \,\mathbf{d}\psi \right)$$

General facts for extremal MKN black hole near the horizon

- Metric has a simple structure involving AdS_2

$$\boldsymbol{g} = \underset{\sim}{f}(\vartheta)\left(-\frac{\chi^2}{\left(Q^2+2a^2\right)^2}\,\mathbf{d}\tau^2 + \frac{\mathbf{d}\chi^2}{\chi^2} + \mathbf{d}\vartheta^2\right) + \\ + \frac{\left(Q^2+2a^2\right)^2 \sin^2\vartheta}{\underset{\sim}{f}(\vartheta)}\left(\mathbf{d}\psi - \tilde{\omega}\chi\,\mathbf{d}\tau\right)^2$$

- Potentials do not depend on latitude

$$\omega = \tilde{\omega}\chi \qquad\qquad \phi = \tilde{\phi}\chi$$

- Electromagnetic field is highly constrained

$$A_\tau = \left(\tilde{N}\sqrt{\tilde{g}_{rr}}F_{(r)(t)}\right)\Big|_{r_0}\chi \equiv \tilde{A}_\tau(\vartheta)\,\chi$$

$$A_\psi(\vartheta) = \frac{1}{\tilde{\omega}}\left(\tilde{A}_\tau(0) - \tilde{A}_\tau(\vartheta)\right) \qquad\qquad \phi = -\left.A_\tau\right|_{\vartheta=0}$$

Near-horizon limit of extremal MKN black hole (raw)

Out[12]=
$$\frac{1}{64\left(a^2+Q^2+a^2\cos[\theta]^2\right)}\left(3a^2+2Q^2+a^2\cos[2\theta]\right)$$
$$\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+$$
$$\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)$$

Out[13]=
$$\frac{32a^4B^3Q+32a^2B^3Q^3+16a^5B^4\sqrt{a^2+Q^2}+16a^3B^4Q^2\sqrt{a^2+Q^2}+4BQ^3\left(4+B^2Q^2\right)+a\sqrt{a^2+Q^2}\left(-16+24B^2Q^2+3B^4Q^4\right)}{8\left(2a^2+Q^2\right)^2}$$

Out[14]=
$$\Big(128a^7B^5\sqrt{a^2+Q^2}+192a^5B^3\sqrt{a^2+Q^2}\left(4+B^2Q^2\right)-2Q^3\left(-2+BQ\right)\left(2+BQ\right)\left(16+B^4Q^4\right)-24a^4B^2Q\left(-48-8B^2Q^2+B^4Q^4\right)+$$
$$8aBQ^2\sqrt{a^2+Q^2}\left(80-16B^2Q^2+3B^4Q^4\right)+8a^3B\sqrt{a^2+Q^2}\left(16+136B^2Q^2+5B^4Q^4\right)+a^2Q\left(64+1232B^2Q^2-148B^4Q^4-9B^6Q^6\right)-$$
$$\Big(64B^2Q^5-16B^4Q^7+128a^7B^5\sqrt{a^2+Q^2}+64a^5B^3\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)-24a^4B^2Q\left(16-24B^2Q^2+B^4Q^4\right)-8aBQ^2\sqrt{a^2+Q^2}\left(16-32B^2Q^2+3B^4Q^4\right)+$$
$$8a^3B\sqrt{a^2+Q^2}\left(16+8B^2Q^2+5B^4Q^4\right)+a^2Q\left(64-304B^2Q^2+236B^4Q^4-9B^6Q^6\right)\Big)\cos[2\theta]-32a^6B^4Q\left(-20+B^2Q^2\right)\sin[\theta]^2\Big)\Big/$$
$$\Big(4\left(2a^2+Q^2\right)\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+$$
$$\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)\Big)$$

Out[15]=
$$-\frac{2B\left(12Q^2+24aBQ\sqrt{a^2+Q^2}+B^2\left(16a^4+16a^2Q^2+Q^4\right)\right)}{16+B^2\left(24Q^2+32aBQ\sqrt{a^2+Q^2}+B^2\left(16a^4+16a^2Q^2+Q^4\right)\right)}+$$
$$\Big(2\Big(48a^6B^3+72a^3B^2Q\sqrt{a^2+Q^2}+2BQ^4\left(8+B^2Q^2\right)+8a^4B\left(2+9B^2Q^2\right)+2aQ\sqrt{a^2+Q^2}\left(4+15B^2Q^2\right)+a^2BQ^2\left(52+27B^2Q^2\right)+$$
$$\left(16a^6B^3+8BQ^4+24a^3B^2Q\sqrt{a^2+Q^2}+8a^4B\left(-2+3B^2Q^2\right)+a^2BQ^2\left(-4+9B^2Q^2\right)+2aQ\sqrt{a^2+Q^2}\left(-4+9B^2Q^2\right)\right)\cos[2\theta]\Big)\Big)\Big/$$
$$\Big(48a^6B^4+96a^3B^3Q\sqrt{a^2+Q^2}+8aBQ\sqrt{a^2+Q^2}\left(4+5B^2Q^2\right)+8a^4B^2\left(4+9B^2Q^2\right)+2Q^2\left(16+16B^2Q^2+B^4Q^4\right)+a^2\left(48+104B^2Q^2+27B^4Q^4\right)+$$
$$\left(16a^6B^4+16B^2Q^4+32a^3B^3Q\sqrt{a^2+Q^2}+8a^4B^2\left(-4+3B^2Q^2\right)+8aBQ\sqrt{a^2+Q^2}\left(-4+3B^2Q^2\right)+a^2\left(16-8B^2Q^2+9B^4Q^4\right)\right)\cos[2\theta]\Big)$$

The equivalence

- The near-horizon description of a MKN black hole can be obtained from the one of the Kerr-Newman solution, using effective parameters:

$$\hat{M} = \sqrt{Q^2 + a^2}\left(1 + \frac{1}{4}B^2Q^2 + B^2a^2\right) + BQa$$

$$\hat{a} = a\left(1 - \frac{3}{4}B^2Q^2 - B^2a^2\right) - BQ\sqrt{Q^2 + a^2}$$

$$\hat{Q} = Q\left(1 - \frac{1}{4}B^2Q^2\right) + 2Ba\sqrt{Q^2 + a^2}$$

- Note that by definition $\hat{M}^2 = \hat{Q}^2 + \hat{a}^2$
- Rescaling of the Killing vectors

$$\Xi = \frac{1}{1 + \frac{3}{2}B^2Q^2 + 2B^3Qa\sqrt{Q^2 + a^2} + B^4\left(\frac{1}{16}Q^4 + Q^2a^2 + a^4\right)}$$

Near-horizon limit of extremal MKN black hole (factorised)

$$\boldsymbol{g} = \underset{\sim}{f}(\vartheta)\left(-\frac{\chi^2}{(Q^2+2a^2)^2}\,\mathbf{d}\tau^2 + \frac{\mathbf{d}\chi^2}{\chi^2} + \mathbf{d}\vartheta^2\right) + \frac{\left(Q^2+2a^2\right)^2\sin^2\vartheta}{\underset{\sim}{f}(\vartheta)}\left(\mathbf{d}\psi - \tilde{\omega}\chi\,\mathbf{d}\tau\right)^2$$

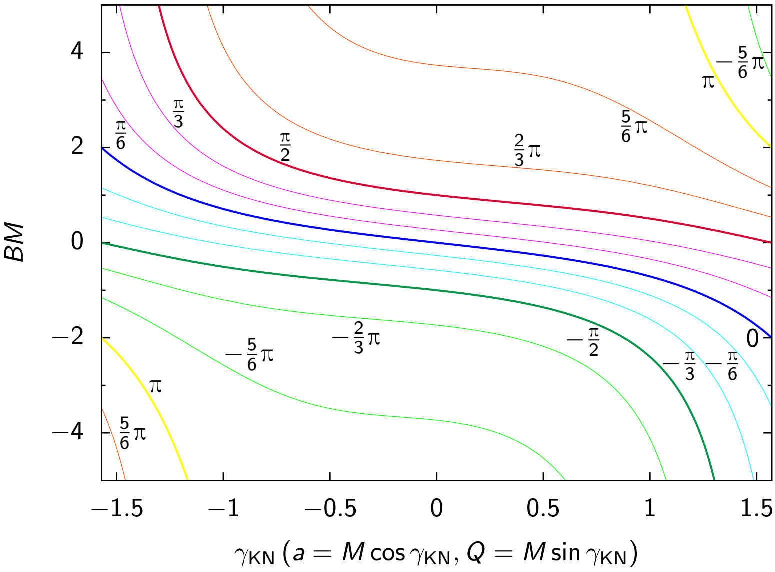
$$\underset{\sim}{f}(\vartheta) = \left[\sqrt{Q^2+a^2}\left(1+\frac{1}{4}B^2Q^2+B^2a^2\right)+BQa\right]^2 + \\ + \left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]^2\cos^2\vartheta$$

$$\tilde{\omega} = -\frac{2}{(Q^2+2a^2)^2}\left[\sqrt{Q^2+a^2}\left(1+\frac{1}{4}B^2Q^2+B^2a^2\right)+ \\ + BQa\right]\left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]$$

$$A_\tau = \frac{1}{\underset{\sim}{f}(\vartheta)}\frac{\chi}{Q^2+2a^2}\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\left\{\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+ \\ + 2Ba\sqrt{Q^2+a^2}\right]^2+\left[a\left(1-\frac{3}{4}B^2Q^2-B^2a^2\right)-BQ\sqrt{Q^2+a^2}\right]^2\sin^2\vartheta\right\}$$

$$A_\psi = -\frac{\tilde{\omega}}{2\underset{\sim}{f}(\vartheta)}\frac{(Q^2+2a^2)^2\left[Q\left(1-\frac{1}{4}B^2Q^2\right)+2Ba\sqrt{Q^2+a^2}\right]\sin^2\vartheta}{1+\frac{3}{2}B^2Q^2+2B^3Qa\sqrt{Q^2+a^2}+B^4\left(\frac{1}{16}Q^4+Q^2a^2+a^4\right)}$$

Parameter space of a near-horizon MKN: $\hat{a} = \hat{M}\cos\hat{\gamma}_{KN}$, $\hat{Q} = \hat{M}\sin\hat{\gamma}_{KN}$



Corollaries, related facts

- General curve describing equivalent geometries

$$BM = \frac{4\cos\gamma_{\mathrm{KN}} - \sin 2\gamma_{\mathrm{KN}}\sin\hat{\gamma}_{\mathrm{KN}} \mp 2\left(1+\cos^2\gamma_{\mathrm{KN}}\right)\cos\hat{\gamma}_{\mathrm{KN}}}{\sin^3\gamma_{\mathrm{KN}} + \left(1+3\cos^2\gamma_{\mathrm{KN}}\right)\sin\hat{\gamma}_{\mathrm{KN}}}$$

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- **Lewandowski**, J., **Pawlowski**, T., *Extremal isolated horizons: A local uniqueness theorem*, Classical Quantum Gravity, 20, 2003.
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- **Gibbons**, G. W. , **Pang**, Yi, **Pope**, C. N. *Thermodynamics of magnetized Kerr-Newman black holes*. Physical Review D, 89, 2014.
- **Booth**, I., **Hunt**, M. *et al.*, *Insights from Melvin–Kerr–Newman spacetimes*, Classical Quantum Gravity, 32, 2015.
- Intuitive angular momentum $\hat{J} \equiv \hat{a}\hat{M}$, $\hat{J} = J_{\mathrm{W}} = J_{\mathrm{IH}}$
- Different masses $M_{\mathrm{KK}} \equiv {}^{M}/_{\Xi}$, $M_{\mathrm{KK}} \neq \hat{M} = M_{\mathrm{IH}}$ (open problem)

Corollaries II, Meissner effect

- Four Killing vectors leading to a Killing tensor
- **Galajinsky**, A., **Orekhov**, K., $\mathcal{N} = 2$ *superparticle near horizon of extreme Kerr–Newman–AdS–dS black hole*, Nuclear Physics B 850, 2011.

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- **Galajinsky**, A., **Orekhov**, K., *$\mathcal{N} = 2$ superparticle near horizon of extreme Kerr–Newman–AdS–dS black hole*, Nuclear Physics B 850, 2011.
- We can substitute for $\hat{a}$

$$\hat{a} = \frac{\operatorname{sgn} \hat{J}}{\sqrt{2}} \sqrt{\sqrt{\hat{Q}^4 + 4\hat{J}^2} - \hat{Q}^2}$$

- Magnetic flux through the upper hemisphere of the horizon

$$\mathscr{F}_{\mathrm{H}} = 2\pi \frac{A_\psi|_{\vartheta=\frac{\pi}{2}}}{\Xi} = 2\pi \frac{\hat{Q}\hat{a}}{\hat{M}} = \frac{4\pi \hat{Q}\hat{J}}{\hat{Q}^2 + \sqrt{\hat{Q}^4 + 4\hat{J}^2}}$$

- **Karas**, V., **Vokrouhlický**, D. *On interpretation of the magnetized Kerr-Newman black hole*. Journal of Mathematical Physics, 32, 714-716, 1991.

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  GAUK No. 606412: "Examination of fields in curved spacetimes"