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Spontaneous scalarization in anisotropic configurations

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Outline

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1. Introduction
   - The importance of neutron stars
   - Why study anisotropy?
2. Framework
   - Scalar-tensor theories
   - Slowly rotating approximation
3. Results
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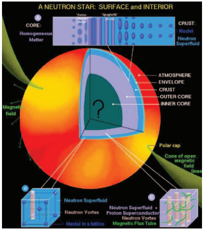
   - Anisotropy in ST

The importance of neutron stars

Neutron stars are ideal objects to test physics:

- Mass $\sim M_\odot$ and radius $\sim 10\mathrm{km}$. Density $\sim 10^{14}\mathrm{g/cm^3}$.
- Neutron star are highly relativistic objects – test gravity!
- Matter in extreme conditions: High pressure, temperature, magnetic fields...

Highly relativistic, quantum matter, with magnetic fields, radiative transfer... This should be complicated to deal.



Credit: D. Page.

Maybe not... Fluid description

The bulk characteristics of NS can be described by a fluid model:

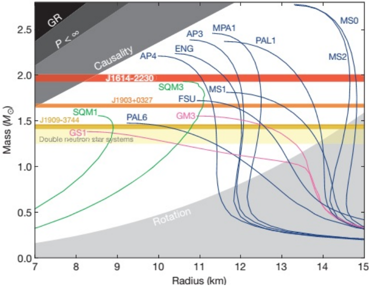
$$T^{ab} = (p+\rho)u^a u^b + p\, g^{ab}.$$

Note that this assumes that the fluid which compose the NS is isotropic.

Einstein equations:

$$G_{ab} = \kappa T_{ab},$$
$$\nabla_a T^{ab} = 0,$$

provided with an EOS $p = p(\rho)$. TOV equations.



Credit: Demorest *et al.*, Nature 2010.

Outline

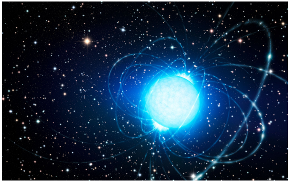
Why study anisotropy?

Anisotropic may be relevant for the properties of NS.

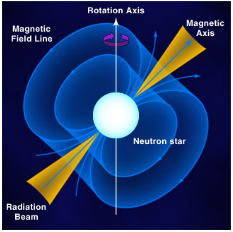
Herrera & Santos, P. Rep. **286** 53, 1997.

- Magnetic fields;
  Yazadjiev, PRD **85** 044030, 2012.
- Nuclear matter at high densities;
  Nelmes & Piette, PRD **85** 123004, 2012.
  Adam *et al.*, PLB **742** 136, 2014.
- Effective models;
  Letelier, PRD **20** 807, 1980.
  P. Boonserm *et al.*, arXiv:1501.07044, 2015.
- Exotic objects;
  Schunck & Mielke, CQG **20** R301, 2003.
  Cattoen *et al.*, CQG **25** 4189, 2005.
- Rotating stars;
  Bayin, PRD **26** 1262, 1982.

Here: Slowly rotating NS stars with anisotropic pressure – Within GR and ST theories.



Credit: ESO/L. Calçada.



Credit: ESO/L. Calçada.

Outline

Scalar-Tensor theories of gravity

Among the simplest and well studied extensions of GR.

$$S = \frac{c^4}{16\pi G_*} \int d^4x \frac{\sqrt{-g_*}}{c} \left(R_* - 2g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi\right) + S_{\rm M}\left[\psi_{\rm M}; A^2(\varphi)g_{*\mu\nu}\right], \tag{1}$$

$$\Downarrow$$

$$R_{*\mu\nu} = 2\partial_\mu\varphi\partial_\nu\varphi + 8\pi\left(T_{*\mu\nu} - \frac{1}{2}T_* g_{*\mu\nu}\right), \tag{2}$$

$$\Box_*\varphi = -4\pi\alpha(\varphi)T_*, \tag{3}$$

where

$$T_*^{\mu\nu} \equiv \frac{2}{\sqrt{-g_*}} \frac{\delta S_M\left[\psi_M, A^2(\varphi)g_{*\mu\nu}\right]}{\delta g_{*\mu\nu}}. \tag{4}$$

Outline

Slowly rotating approximation

First order in rotation

Hartle, APJ **150** 1005, 1967, and Hartle & Thorne, APJ **153** 807, 1068.

We shall take $A(\varphi) = \exp(\beta\varphi^2/2)$. The metric is

$$d\tilde{s}^2 = A^2(\varphi)\Big[- e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2 - 2\,\omega(\mathrm{r},\theta)\mathrm{r}^2 \sin^2\theta \, \mathrm{dt}\,\mathrm{d}\phi\Big], \tag{5}$$

where

$$e^{-2\Lambda(r)} \equiv 1 - \frac{2\mu(r)}{r}, \tag{6}$$

and

$$\tilde{u}^\mu = A^{-1}(\varphi)\left(e^{-\Phi}, 0, 0, \Omega\, e^{-\Phi}\right). \tag{7}$$

With this, assuming the an explicity form for the stress-energy tensor, we obtain the modified TOV equations.

Anisotropic matter

The energy-momentum tensor

In the Jordan frame, we have

$$\boxed{\tilde{T}_{\mu\nu} = \tilde{\epsilon}\,\tilde{u}_\mu \tilde{u}_\nu + \tilde{p}\,\tilde{k}_\mu \tilde{k}_\nu + \tilde{q}\,\tilde{\Pi}_{\mu\nu},}$$

where $\tilde{u}_\mu$ is the fluid four-velocity, $\tilde{k}_\mu$ is a unit radial vector (i.e. $\tilde{k}_\mu \tilde{k}^\mu = 1$), satisfying $\tilde{u}^\mu \tilde{k}_\mu = 0$ and $\tilde{\Pi}_{\mu\nu} \equiv \tilde{g}_{\mu\nu} + \tilde{u}_\mu \tilde{u}_\nu - \tilde{k}_\mu \tilde{k}_\nu$.

Anisotropic models

$$\tilde{\sigma} \equiv \tilde{p} - \tilde{q}.$$

Quasi-local EOS:

Horvat et al., Class.Quant.Grav. **28** 025009, 2015.

$$\tilde{\sigma} \equiv \lambda_{\rm H} \tilde{p} \frac{2\mu}{r}.$$

Bowers-Liang model:

Bowers and Liang, ApJ **188** 657, 1974.

$$\tilde{\sigma} \equiv \frac{1}{3} \lambda_{\rm BL} \left(\tilde{\epsilon} + 3\tilde{p}\right) \left(\tilde{\epsilon} + p\right) \left(1 - \frac{2\mu}{r}\right)^{-1} r^2.$$

The modified TOV equations

$$\frac{d\mu}{dr} = 4\pi A^4(\varphi) r^2 \tilde{\epsilon} + \frac{1}{2} r(r-2\mu)\psi^2, \quad (8)$$

$$\frac{d\Phi}{dr} = 4\pi A^4(\varphi) \frac{r^2 \tilde{p}}{r-2\mu} + \frac{1}{2} r\psi^2 + \frac{\mu}{r(r-2\mu)}, \quad (9)$$

$$\frac{d\psi}{dr} = 4\pi A^4(\varphi) \frac{r}{r-2\mu} \left[\alpha(\varphi)(\tilde{\epsilon}-3\tilde{p}) + r(\tilde{\epsilon}-\tilde{p})\psi\right] - \frac{2(r-\mu)}{r(r-2\mu)}\psi + 8\pi A^4(\varphi)\alpha(\varphi)\frac{r\tilde{\sigma}}{r-2\mu}, \quad (10)$$

$$\frac{d\tilde{p}}{dr} = -(\tilde{\epsilon}+\tilde{p})\left[\frac{d\Phi}{dr} + \alpha(\varphi)\psi\right] - 2\tilde{\sigma}\left[\frac{1}{r} + \alpha(\varphi)\psi\right], \quad (11)$$

$$\frac{d\varpi}{dr} = 4\pi A^4(\varphi) \frac{r^2}{r-2\mu}(\tilde{\epsilon}+\tilde{p})\left(\varpi + \frac{4\bar{\omega}}{r}\right) + \left(r\psi^2 - \frac{4}{r}\right)\varpi + 16\pi A^4(\varphi)\frac{r\tilde{\sigma}}{r-2\mu}\bar{\omega}. \quad (12)$$

Boundary conditions

We integrate the equations from the origin, requiring regularity. For a large r, we match with:

$$\mu(r) = M - \frac{Q^2}{2r} - \frac{MQ^2}{2r^2} + \mathcal{O}(r^{-3}) \tag{13}$$

$$e^{2\Phi} = 1 - \frac{2M}{r} + \mathcal{O}(r^{-3}), \tag{14}$$

$$\varphi(r) = \varphi_\infty + \frac{Q}{r} + \frac{MQ}{r^2} + \mathcal{O}(r^{-3}), \tag{15}$$

$$\bar{\omega}(r) = \Omega - \frac{2J}{r^3} + \mathcal{O}(r^{-4}). \tag{16}$$

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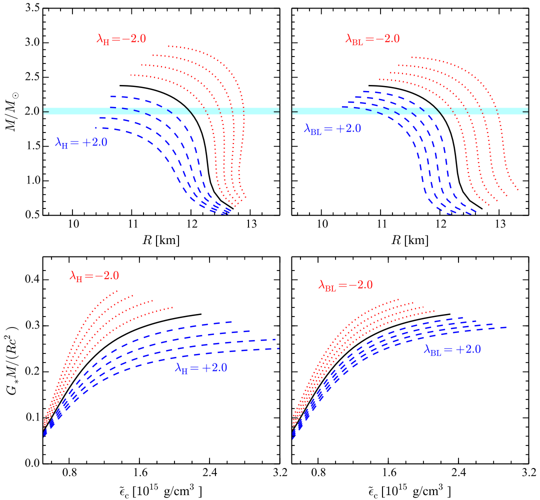
Results

The effect of anisotropy in GR: Mass-radius relation.



Mass-radius relation (top panels) and dimensionless compactness $G_* M/Rc^2$ as a function of the central density (bottom panels) for anisotropic stars in GR using EoS APR. In the left panels we use the quasi-local model; in the right panels, the Bowers-Liang model.

Results

The effect of anisotropy in GR: Moment of inertia.

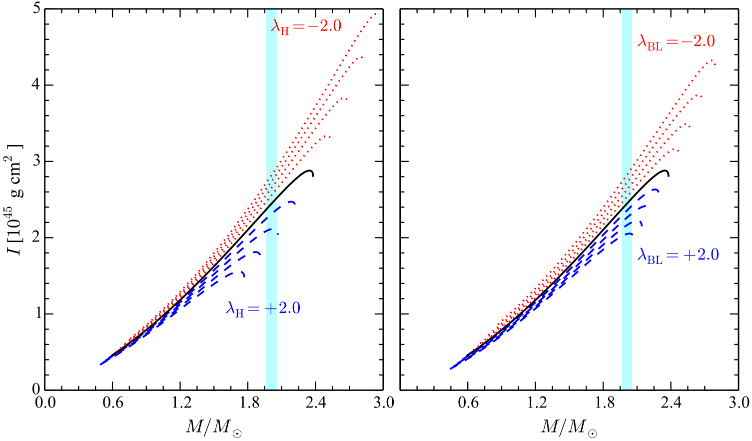


Figure: The moment of inertia I as function of the mass M for anisotropic stars i n GR using EoS APR, increasing $\lambda_{\rm H}$ (or $\lambda_{\rm BL}$) in increments of 0.5 between -2 (top curves) and 2 (bottom curves).

Outline

Anisotropy in ST

Spontaneous scalarization

Damour & Esposito-Farèse, PRL **70** 2220, 1993, and Damour & Esposito-Farèse, PRD **54** 1474, 1996.

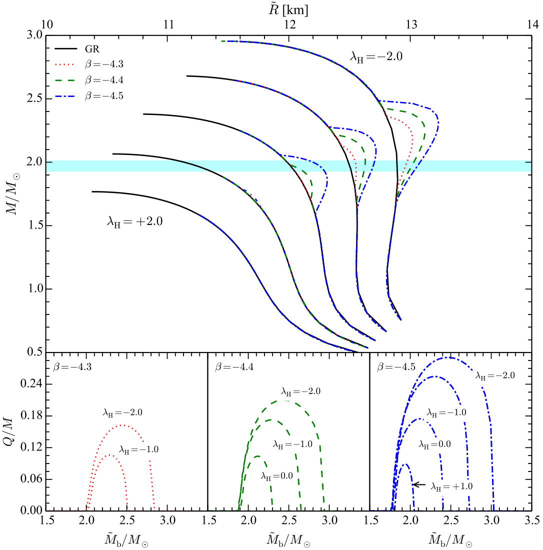
Assuming $A(\varphi) = \exp(\beta\varphi^2/2)$, and $\varphi_\infty = 0$, NS structures can be very different from the GR counterpart. Depending on β and the star structure, the scalar field may acquire a charge. This is called spontaneous scalarization.

- Harada found that for spontaneous scalarization to occur in isotropic configuration $\beta \lesssim -4.35$.
  Harada, PRD **57** 4802, 1998.
- Doneva *et al* showed that if the star rotates scalarization happens for $\beta > -4.35$.
  Doneva *et al.*, PRD **88** 084060, 2013.
- Binary pulsar observations require $\beta \gtrsim -4.5$.
  Freire *et al.*, MNRAS **423** 3328, 2012.

Can spontaneous scalarization be enhanced if the matter is anisotropic?

Anisotropy in ST

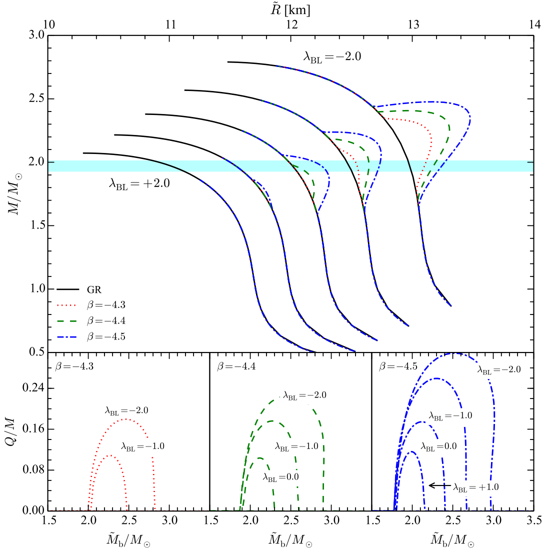
The effect in spontanous scalarization



Spontaneous scalarization in the quasi-local model.

Anisotropy in ST

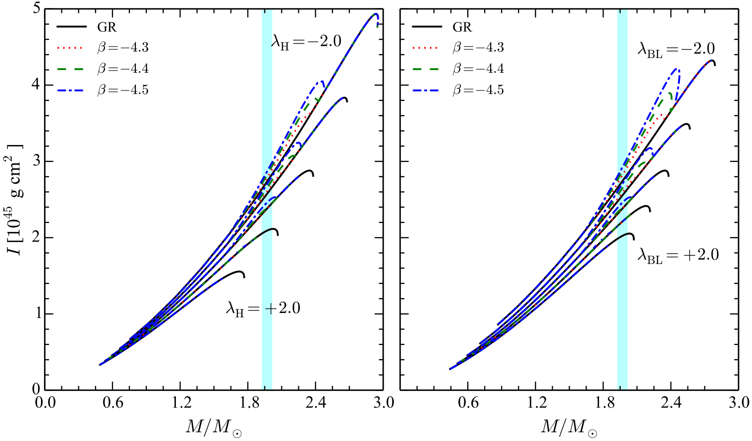
The effect in spontanous scalarization



Spontaneous scalarization in the Bowers-Liang model.

Anisotropy in ST

The effect in spontanous scalarization



Spontaneous scalarization into the moment of inertia.

Anisotropy in ST

Critical scalarization point

We can make a linear approximation in the scalar field to find the threshold for scalarization. Redefining the scalar field as $\varphi(t,r) = r^{-1}\Psi(r)$, we have Schrödinger-like equation:

$$\frac{d^2\Psi}{dx^2} + \left[V_{\rm eff}(x)\right]\Psi = 0, \tag{17}$$

where effective potential is

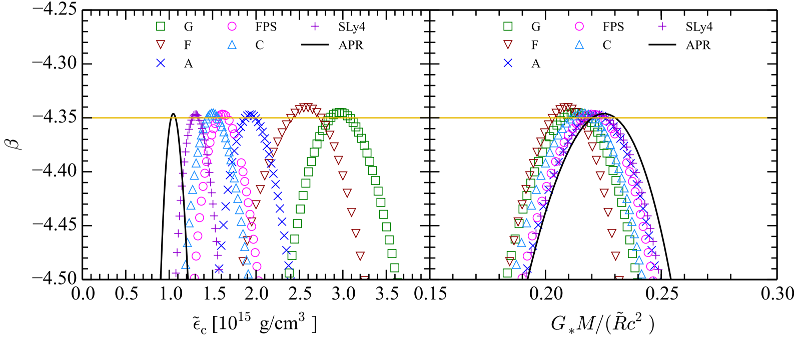
$$V_{\rm eff}(r) \equiv e^{2\Phi}\left[\mu_{\rm eff}^2(r) + \frac{2\mu}{r^3} + 4\pi(\tilde{p} - \tilde{\epsilon})\right], \tag{18}$$

where we have introduced an effective (position-dependent) mass

$$\mu_{\rm eff}^2(r) \equiv -4\pi\beta T_*. \tag{19}$$

Anisotropy in ST

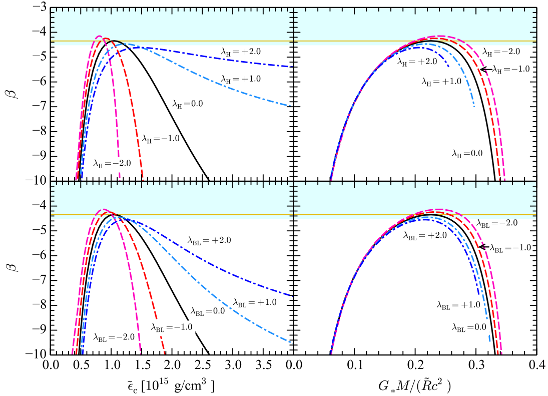
Critical scalarization point



Critical β for scalarization as a function of the central density (left panel) and of the stellar compactness (right panel) for nonrotating NS models constructed using different nuclear-physics based EoSs, in the absence of anisotropy.

Anisotropy in ST

Critical scalarization point



Left panels: β versus critical central densities for different values of $\lambda_{\rm H,BL}$. Right panels: β versus compactness $G_*M/\tilde{R}c^2$ of the critical solutions for different values of $\lambda_{\rm H,BL}$.

$\beta_{\rm max} \sim -4.15$ and -4.13!

Summary

- Anisotropy have a big influence in the mass-radius curves and in the moment of inertia, even for GR.
- It strongly affects the spontaneous scalarization in neutron stars.
- Observation of binary pulsars with $\beta > -4.35$ would be a strong evidence for anisotropy.
- The work can be extended (and should) to microphysics anisotropic models.
- It would be interesting to identify exclusion regions in the (β, λ) parameter space.

THANK YOU!