

# The Universe after *Planck* 2013

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for the PLANCK Collaboration

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# Outline

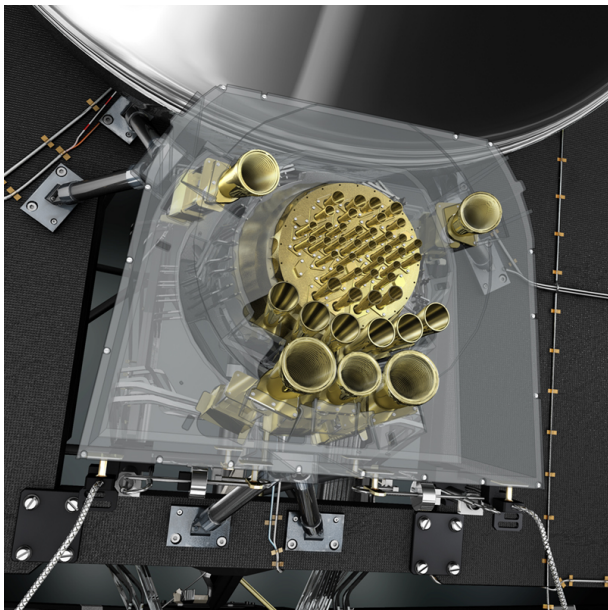
1. What is Planck?
2. A brief history of CMB observation
3. First release of cosmology results
4. Power spectrum
5. Gravitational lensing
6. Non-Gaussianity
7. Statistical isotropy
8. Alignments
9. Clusters
10. What's next

What is Planck?

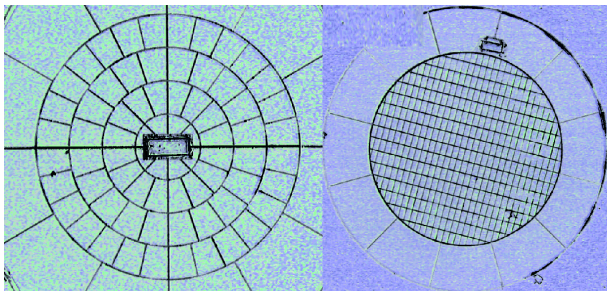
## The *Planck* mission



# PLANCK Focal Plane

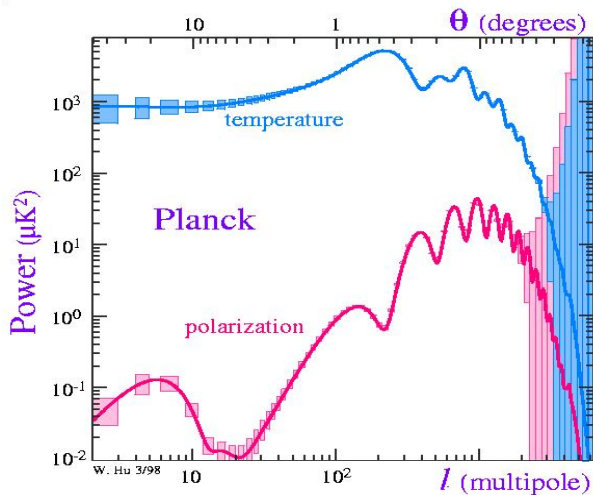


# The workhorse of Planck: spiderweb and polarization sensitive bolometers



Made by JPL, Caltech  
Cooled to  $\approx 100\text{mK}$

# Planck Capabilities

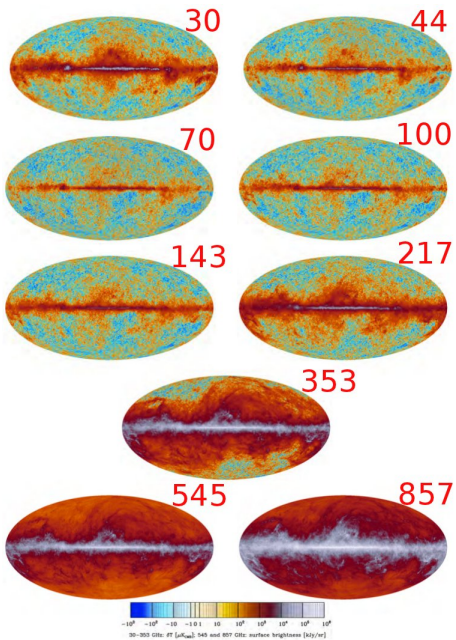


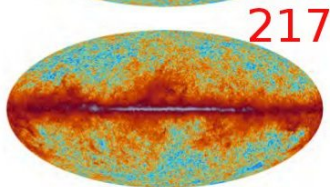
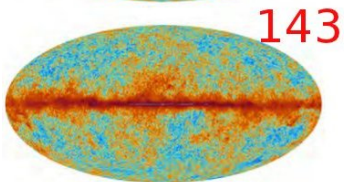
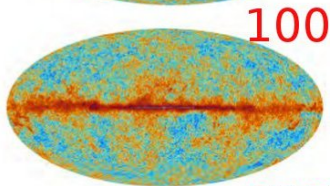
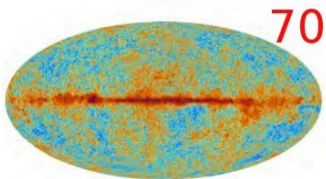
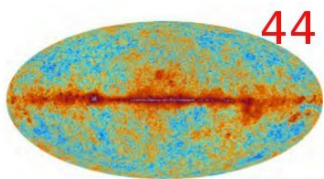
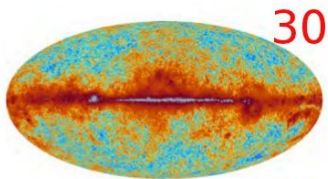
# Planck Capabilities

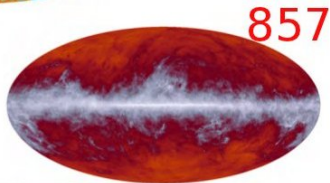
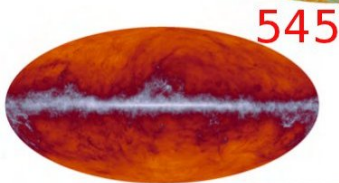
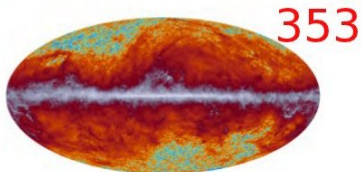
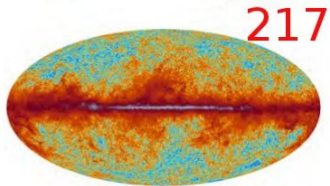
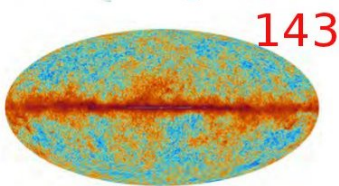
**Table 2.** *Planck* performance parameters determined from flight data.

| CHANNEL       | $N_{\text{detectors}}^{\text{a}}$ | $\nu_{\text{center}}^{\text{b}}$<br>[GHz] | SCANNING BEAM <sup>c</sup> |             | NOISE <sup>d</sup><br>SENSITIVITY  |                     |
|---------------|-----------------------------------|---|----------------------------|-------------|--|---------------------|
|               |                                   |   | FWHM<br>[arcmin]           | Ellipticity | [ $\mu\text{K}_{\text{RJ}} \text{s}^{1/2}$ ] [ $\mu\text{K}_{\text{CMB}} \text{s}^{1/2}$ ] |                     |
| 30 GHz .....  | 4                                 | 28.4                                      | 33.16                      | 1.37        | 145.4  | 148.5               |
| 44 GHz .....  | 6                                 | 44.1                                      | 28.09                      | 1.25        | 164.8  | 173.2               |
| 70 GHz .....  | 12                                | 70.4                                      | 13.08                      | 1.27        | 133.9  | 151.9               |
| 100 GHz ..... | 8                                 | 100                                       | 9.59                       | 1.21        | 31.52  | 41.3                |
| 143 GHz ..... | 11                                | 143                                       | 7.18                       | 1.04        | 10.38  | 17.4                |
| 217 GHz ..... | 12                                | 217                                       | 4.87                       | 1.22        | 7.45   | 23.8                |
| 353 GHz ..... | 12                                | 353                                       | 4.7                        | 1.2         | 5.52   | 78.8                |
| 545 GHz ..... | 3                                 | 545                                       | 4.73                       | 1.18        | 2.66   | 0.0259 <sup>d</sup> |
| 857 GHz ..... | 4                                 | 857                                       | 4.51                       | 1.38        | 1.33   | 0.0259 <sup>d</sup> |

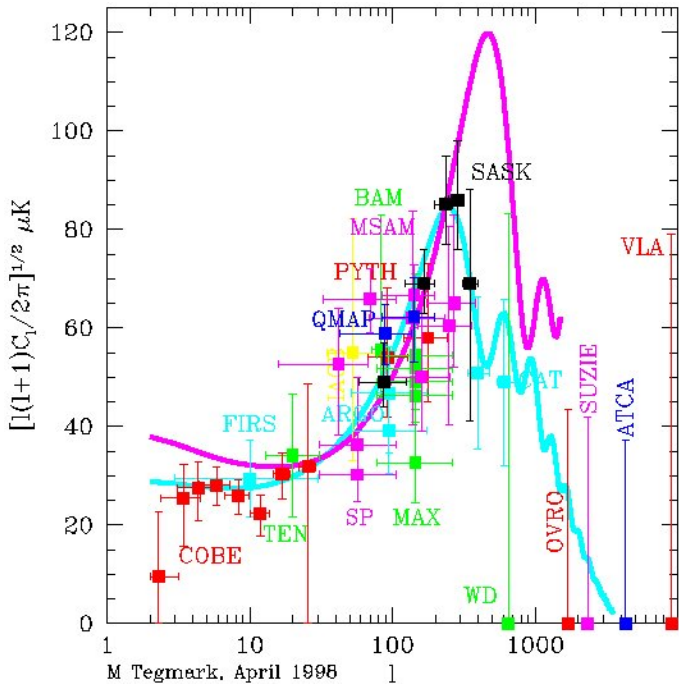








## A brief history of CMB observation

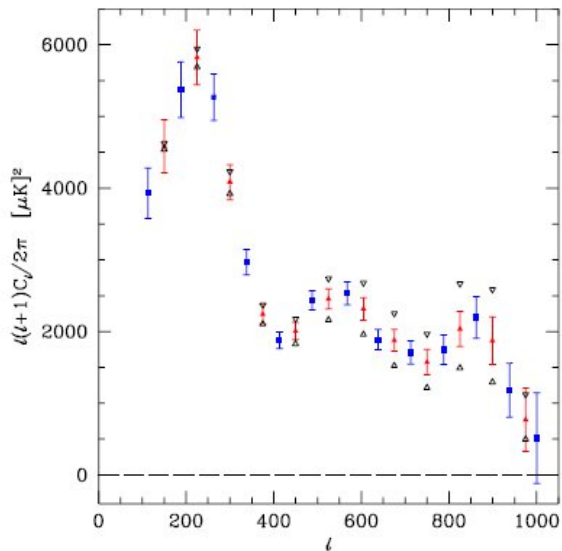


M Tegmark, April 1996

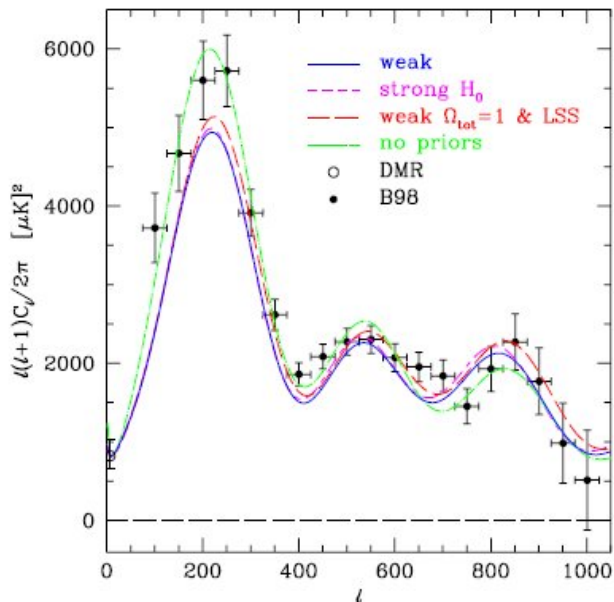
# Boomerang baloon



# Boomerang mutipole spectrum (no theory)

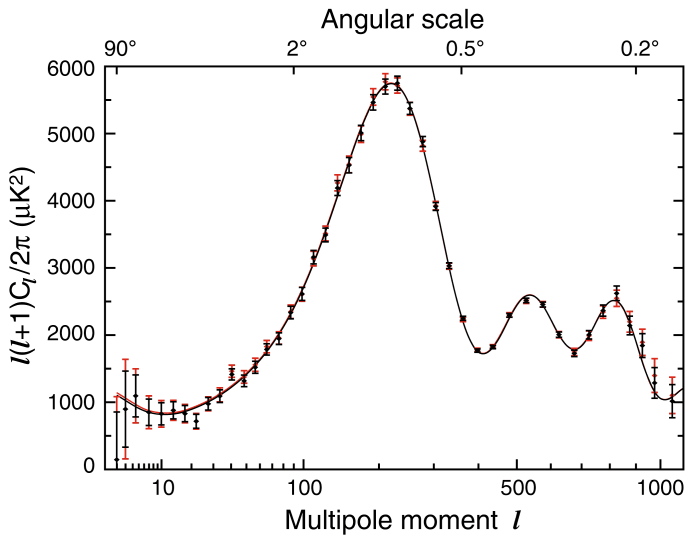


# Boomerang multipole spectrum (with theory)

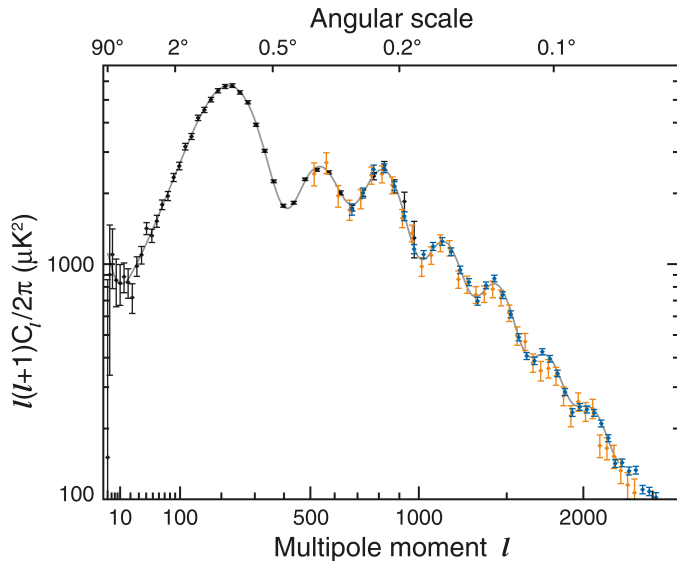




# WMAP 9-year alone



# WMAP 9-year + SPT + ACT



## Basic statistical approach—interesting outcome case

1. No popular model provides good fit to data.
2. Former agreement with “concordance” model (sufficient to explain WMAP power spectrum and the loose constraints at very large  $\ell$  from ACT and SPT) falls apart owing to factor 2 improvement in resolution and factor 10 improvement in sensitivity of Planck.
3. No simple, theoretically motivated model works, although baroque models with epicycles and many parameters can be made to work.
4. Theorists sent back to drawing board.

This did not happen, but polarization data still being analyzed. Still room for some surprises in future releases.

## Basic statistical approach—boring outcome case

1. After having constructed a **likelihood function** whose input is the predicted theoretical power spectrum, find the simplest model with a **good fit** to the power spectrum.
2. Consider extensions to this model and see whether the improvement in the quality of fit is statistically significant. (E.g., isocurvature modes, extra neutrino species, varying  $\alpha$ , ...)
3. Study the residuals to the minimal models to test for statistical significance.

# The Planck Temperature-Temperature Power Spectrum $C_{TT}(\ell)$

# Base model—sampling parameters

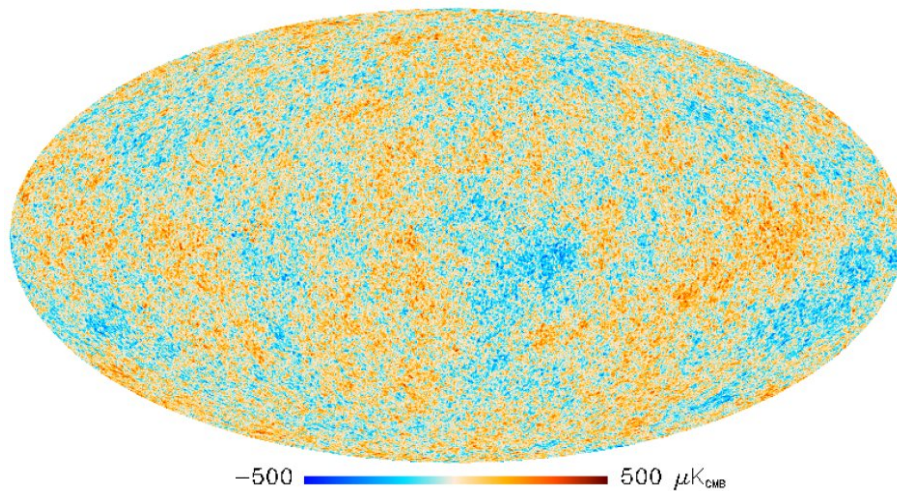
| Parameter                    | <i>Planck</i> |                       | <i>Planck+lensing</i> |                       | <i>Planck+WP</i> |                           |
|------------------------------|---------------|-----------------------|-----------------------|-----------------------|------------------|---------------------------|
|                              | Best fit      | 68% limits            | Best fit              | 68% limits            | Best fit         | 68% limits                |
| $\Omega_b h^2$ . . . . .     | 0.022068      | $0.02207 \pm 0.00033$ | 0.022242              | $0.02217 \pm 0.00033$ | 0.022032         | $0.02205 \pm 0.00028$     |
| $\Omega_c h^2$ . . . . .     | 0.12029       | $0.1196 \pm 0.0031$   | 0.11805               | $0.1186 \pm 0.0031$   | 0.12038          | $0.1199 \pm 0.0027$       |
| $100\theta_{MC}$ . . . . .   | 1.04122       | $1.04132 \pm 0.00068$ | 1.04150               | $1.04141 \pm 0.00067$ | 1.04119          | $1.04131 \pm 0.00063$     |
| $\tau$ . . . . .             | 0.0925        | $0.097 \pm 0.038$     | 0.0949                | $0.089 \pm 0.032$     | 0.0925           | $0.089^{+0.012}_{-0.014}$ |
| $n_s$ . . . . .              | 0.9624        | $0.9616 \pm 0.0094$   | 0.9675                | $0.9635 \pm 0.0094$   | 0.9619           | $0.9603 \pm 0.0073$       |
| $\ln(10^{10} A_s)$ . . . . . | 3.098         | $3.103 \pm 0.072$     | 3.098                 | $3.085 \pm 0.057$     | 3.0980           | $3.089^{+0.024}_{-0.027}$ |

## Base model—derived parameters

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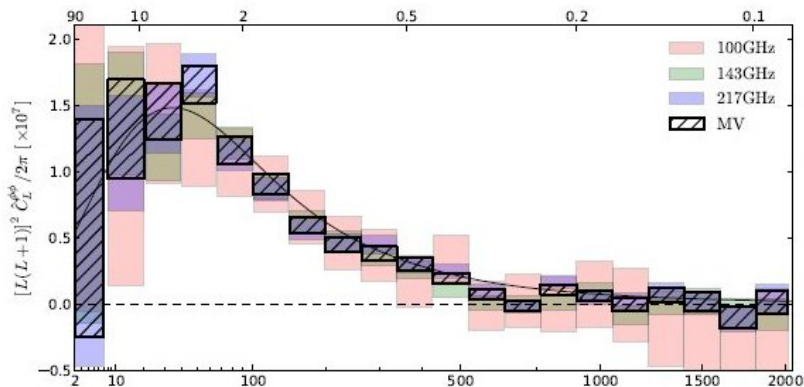
|                            |          |                       |
|----------------------------|----------|-----------------------|
| $\Omega_\Lambda$ . . . . . | 0.6825   | $0.686 \pm 0.020$     |
| $\Omega_m$ . . . . .       | 0.3175   | $0.314 \pm 0.020$     |
| $\sigma_8$ . . . . .       | 0.8344   | $0.834 \pm 0.027$     |
| $z_{\text{re}}$ . . . . .  | 11.35    | $11.4^{+4.0}_{-2.8}$  |
| $H_0$ . . . . .            | 67.11    | $67.4 \pm 1.4$        |
| $10^9 A_s$ . . . . .       | 2.215    | $2.23 \pm 0.16$       |
| $\Omega_m h^2$ . . . . .   | 0.14300  | $0.1423 \pm 0.0029$   |
| $\Omega_m h^3$ . . . . .   | 0.09597  | $0.09590 \pm 0.00059$ |
| $Y_p$ . . . . .            | 0.247710 | $0.24771 \pm 0.00014$ |
| Age/Gyr . . . . .          | 13.819   | $13.813 \pm 0.058$    |

# Planck ILC map

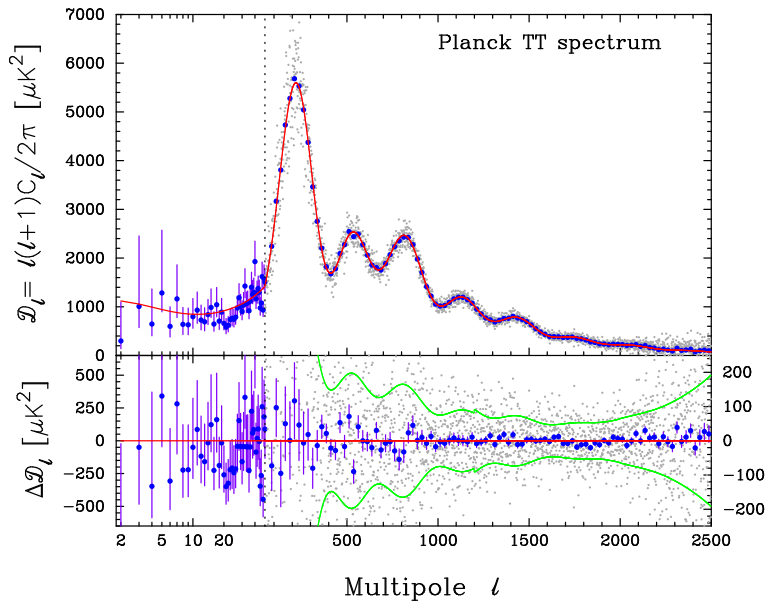


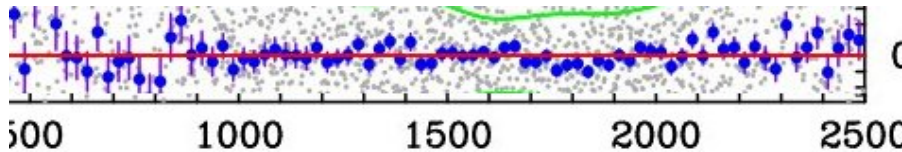
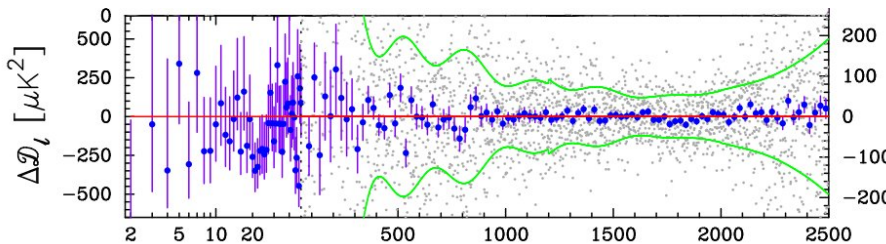


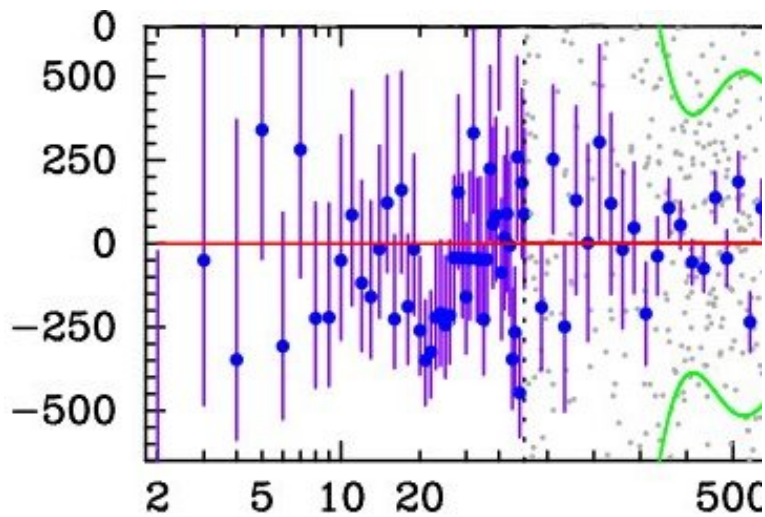
# Planck Gravitational lensing spectrum



**Fig. 18.** Fiducial lensing power spectrum estimates based on the 100, 143, and 217 GHz frequency reconstructions, as well as the minimum-variance reconstruction that forms the basis for the *Planck* lensing likelihood.







## Underlying question: conventional parameterization

### What is the primordial power spectrum?

- ▶ For lack of a fundamental theory, expand in powers of  $\ln(k)$

$$\begin{aligned}\ln(\mathcal{P}(\ln k)) &= \mathcal{P}_0 \left( \ln(k/k_{piv}) \right)^0 + \mathcal{P}_1 \left( \ln(k/k_{piv}) \right)^1 + \mathcal{P}_2 \left( \ln(k/k_{piv}) \right)^2 + \dots \\ \mathcal{P}(k) &= A(k/k_{piv})^{(n_s-1)} \\ \text{or} \\ \mathcal{P}(k) &= A(k/k_{piv})^{(n_s-1)+\alpha \ln(k/k_{piv})+\dots}\end{aligned}$$

- ▶ *Planck* seems to be telling us that the first two terms suffice, and using just the first term can be ruled out at a respectable statistical significance.  $n_s \neq 1$  implies exact scale invariance needs to be downgraded to an approximate symmetry. No statistically significant evidence for running of the spectral index.

# Underlying question: searching for features

- ▶ Two approaches
  - ▶ Parameterized approaches : make Ansätze with a small number of extra parameters and compare quality of fit to simpler model to determine whether extra parameters are justified by the data (Aikake Information Criterion, Bayesian Information Criterion, Bayesian Evidence, ...). (Approach followed in Planck paper XXII, section 8)
  - ▶ Non-parameterized approaches: penalized likelihoods,....  
[Details of approach followed in Planck XXII paper follow: Gauthier, Christopher; Bucher, Martin; Reconstructing the primordial power spectrum from the CMB, JCAP 10, 050 (2012) (arXiv:1209.2147) (Approach followed in Planck paper XXII, section 7)]

# Tentative conclusion of Planck Parameters paper

*To the extremely high accuracy afforded by the Planck data, the power spectrum at high multipoles is compatible with the predictions of the base six parameter  $\Lambda$ CDM cosmology. This is the main result of this paper. Fig. 1 does, however, suggest that the power spectrum of the best-fit base  $\Lambda$ CDM cosmology has a higher amplitude than the observed power spectrum at multipoles  $\ell \lesssim 30$ . We will return to this point in Sect. 7.*

- ▶ Conclusion based on looking at overall  $\chi^2$  with a very large number of degrees of freedom.
- ▶ We want to examine whether this conclusion is really justified.

# Penalized likelihood

Let  $\mathcal{P}_0(k) = A_s(k/k_*)^{n_s-1}$  be the best fit power spectrum of the six parameter model. We define a general Ansatz for the power spectrum in terms of a fractional variation,  $f(k)$ , relative to this fiducial model, so that

$$\mathcal{P}_{\mathcal{R}}(k) = \mathcal{P}_0(k) [1 + f(k)]. \quad (1)$$

Any features are then described in terms of  $f(k)$ .

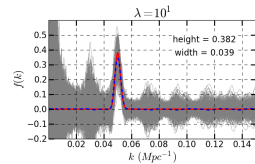
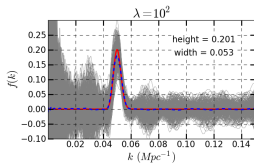
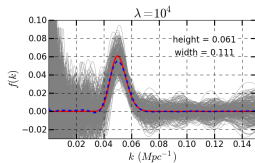
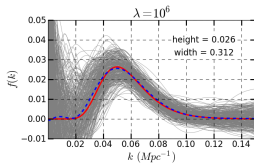
In this analysis we use the Planck+WP likelihood supplemented by the following prior, which is added to  $-2 \ln \mathcal{L}$ :

$$\begin{aligned} \mathbf{f}^T \mathbf{R}(\lambda, \alpha) \mathbf{f} = & \lambda \int d\kappa \left( \frac{\partial^2 f(\kappa)}{\partial \kappa^2} \right)^2 \\ & + \alpha \int_{-\infty}^{\kappa_{\min}} d\kappa f^2(\kappa) + \alpha \int_{\kappa_{\max}}^{+\infty} d\kappa f^2(\kappa). \end{aligned} \quad (2)$$

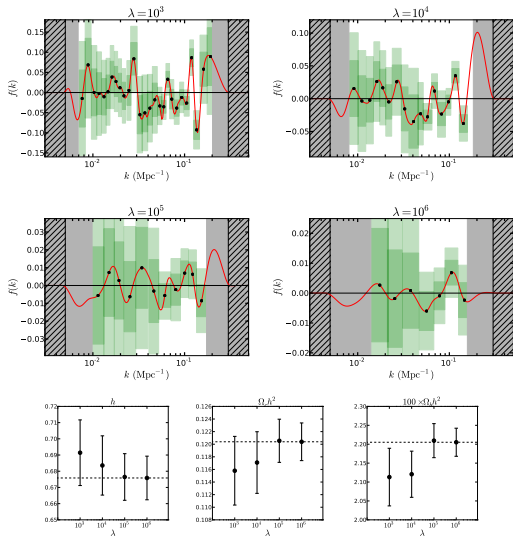
where  $\kappa = \ln k$ .



# Validation of method

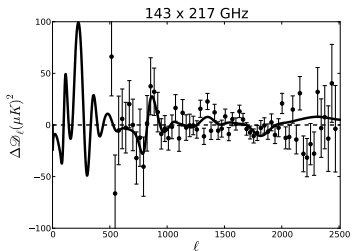
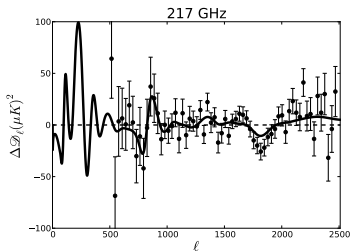
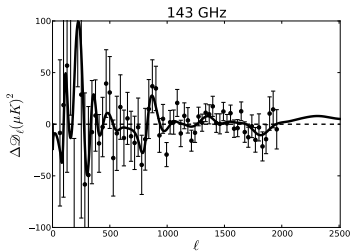
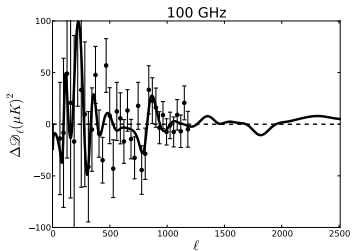


# Results on Planck “Nominal mission” likelihood

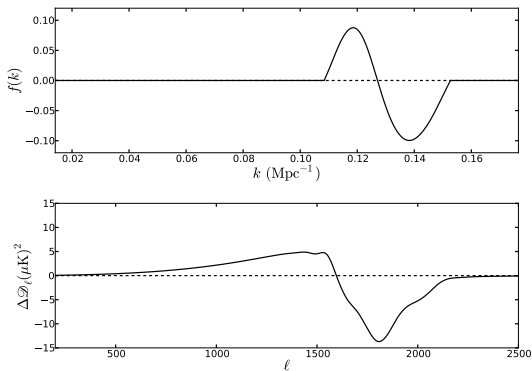


Maximum excursions locally  $3.2\sigma$  and  $3.9\sigma$  for  $\lambda = 10^4$  and  $10^3$ , respectively. After look-elsewhere-effect translates into  $p = 1.74\%$  and  $p = 0.21\%$ , or  $2.4\sigma$  and  $3.1\sigma$ .

Where does this come from in the CMB multipole power spectrum?



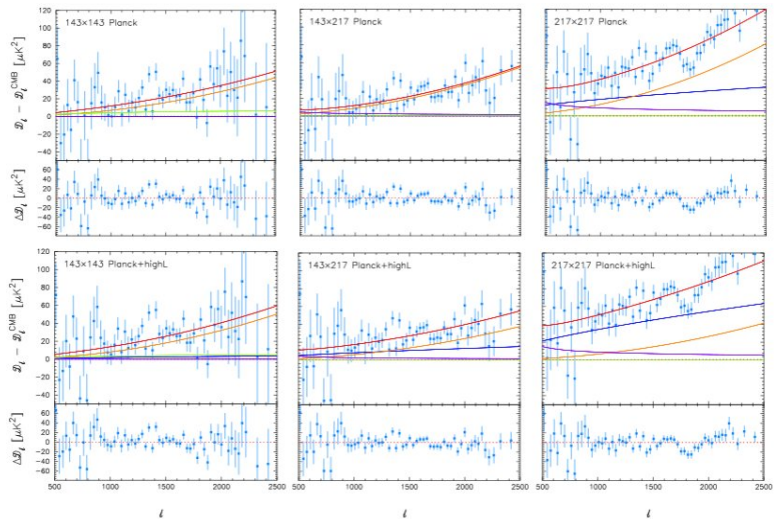
# Proof that signal is from around $\ell \approx 1800$



## (Extract from parameters paper)

*To the extremely high accuracy afforded by the Planck data, the power spectrum at high multipoles is compatible with the predictions of the base six parameter  $\Lambda$ CDM cosmology. This is the main result of this paper. Fig. 1 does, however, suggest that the power spectrum of the best-fit base  $\Lambda$ CDM cosmology has a higher amplitude than the observed power spectrum at multipoles  $\ell \lesssim 30$ . We will return to this point in Sect. 7.*

# (Extract from parameters paper)



# (Extract from parameters paper)

**Table 6.** Goodness-of-fit tests for the *Planck* spectra. The  $\Delta\chi^2 = \chi^2 - N_\ell$  is the difference from the mean assuming the model is correct, and the last column expresses  $\Delta\chi^2$  in units of the dispersion  $\sqrt{2N_\ell}$ .

| Spectrum         | $\ell_{\min}$ | $\ell_{\max}$ | $\chi^2$ | $\chi^2/N_\ell$ | $\Delta\chi^2/\sqrt{2N_\ell}$ |
|------------------|---------------|---------------|----------|-----------------|-------------------------------|
| $100 \times 100$ | 50            | 1200          | 1158     | 1.01            | 0.14                          |
| $143 \times 143$ | 50            | 2000          | 1883     | 0.97            | -1.09                         |
| $217 \times 217$ | 500           | 2500          | 2079     | 1.04            | 1.23                          |
| $143 \times 217$ | 500           | 2500          | 1930     | 0.96            | -1.13                         |
| All              | 50            | 2500          | 2564     | 1.05            | 1.62                          |

Comic interlude: testing for nontrivial topology





**Stephen Hawking** : Your theory of a donut shaped universe is intriguing, Homer. I may have to steal it.

**Homer Simpson** : Wow, I can't believe someone I never heard of is hanging out with a guy like me.

## Homer Simpson's suggestion that the universe is doughnut shaped may not be that far from the truth!

 Like ← 25

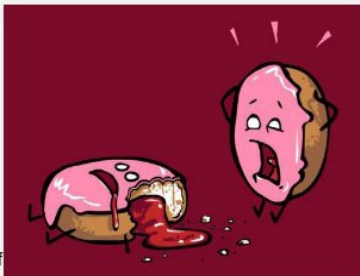
The universe may accidentally be paying tribute to "The Simpsons" with its shape.

The idea that the universe is finite and relatively small rather than infinitely large first became popular in 2003 when cosmologists noticed unexpected patterns in the cosmic microwave background- radiation residue left from the Big Bang.

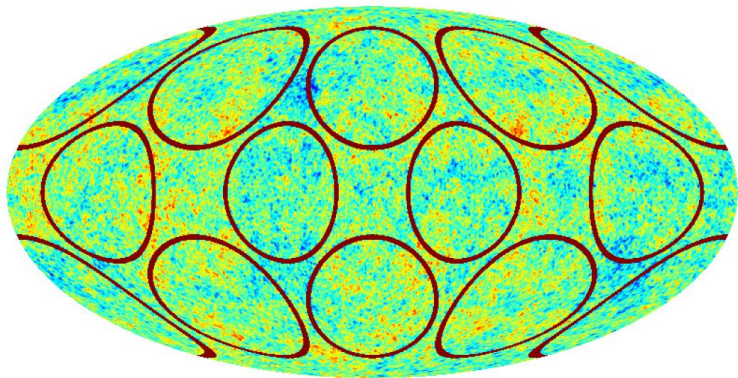
An infinite universe should contain CMB waves of all sizes but cosmologists found that longer wavelengths were missing from measurements made by NASA's Wilkinson Microwave Anisotropy Probe.

This meant that the universe could be finite. When cosmologists set out to determine the shape of this finite universe, it was suggested that the universe could be doughnut shaped.

While the doughnut is a likely candidate, there are some other possibilities (e.g. the universe may be shaped like a football).



## Planck tests Homer's theory—overlapping great circles



**Prediction** : Up to noise and some smearing coincidences on overlapping circles should be apparent.

**Planck result** : These coincidences are not observed ruling out a large class of models with nontrivial topology.

## Constraints on neutrino physics

# Impact of neutrinos on CMB

1. Actual (precision) analysis is somewhat mindless. (1) Incorporate new physics in Boltzmann solver (e.g., CAMB, CLASS,...) (2) Run Monte Carlo Markov Chain (MCMC) (2) Check for convergence, prior dependence, ..... (3) Compare quality of fits.
2. Precision approach gives no intuition. Difficult to understand what is really being tested. Difficult to know what new models to look for in order to explain possible anomalies.
3. Neutrinos affect model CMB primarily in three ways: (1) number of relativistic species shifts moment of matter-radiation equality, affecting the damping tail. (Early recombination means more “viscosity”), (2) “early” integrated Sachs-Wolfe effect, extending visibility surface, (3) gravitational lensing (aka late-time integrated Sachs-Wolfe at large  $\ell$ ). Deficit of halo structure on small-scales.
4. While non-cosmological neutrino experiments depend sensitively on the coupling of neutrinos to the Standard Model, cosmological probes are sensitive to neutrino because of the gravitational coupling. Thus “active” and “sterile” neutrinos imprint the same CMB anisotropies.

# Planck neutrino constraints

WMAP Seven-year claim:

$$N_{eff} = 4.34 \pm 0.87$$

WMAP Nine-year claim:

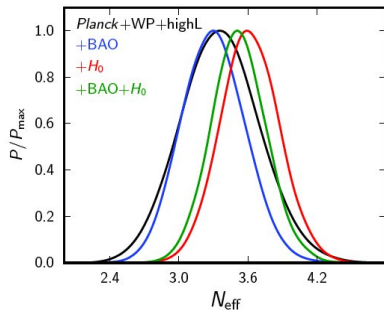
$$N_{eff} = 3.84 \pm 0.40$$

$$\sum m_\nu < 0.44 \text{eV} \text{ 95\% confidence.}$$

Planck:

$$N_{eff} = 3.52^{+0.48}_{-0.45} \text{ (95\% Confidence; Planck + WP + } H_0 \text{ + BAO)}$$

$$\sum m_\nu < 0.28 \text{eV} \text{ 95\% confidence.}$$



## Evidence of Primordial Non-Gaussianity ( $f_{\text{NL}}$ ) in the Wilkinson Microwave Anisotropy Probe 3-Year Data at $2.8\sigma$

Amit P. S. Yadav<sup>1</sup> and Benjamin D. Wandelt<sup>1,2</sup>

<sup>1</sup>*Department of Astronomy, University of Illinois at Urbana-Champaign, 1002 W. Green Street, Urbana, Illinois 61801, USA*

<sup>2</sup>*Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green Street, Urbana, Illinois 61801, USA*

(Received 7 December 2007; revised manuscript received 6 March 2008; published 7 May 2008)

We present evidence for primordial non-Gaussianity of the local type ( $f_{\text{NL}}$ ) in the temperature anisotropy of the cosmic microwave background. Analyzing the bispectrum of the Wilkinson Microwave Anisotropy Probe 3-year data up to  $\ell_{\text{max}} = 750$  we find  $27 < f_{\text{NL}} < 147$  (95% C.L.). This amounts to a rejection of  $f_{\text{NL}} = 0$  at  $2.8\sigma$ , disfavoring canonical single-field slow-roll inflation. The signal is robust to variations in  $\ell_{\text{max}}$ , frequency and masks. No known foreground, instrument systematic, or secondary anisotropy explains it. We explore the impact of several analysis choices on the quoted significance and find  $2.5\sigma$  to be conservative.

DOI: [10.1103/PhysRevLett.100.181301](https://doi.org/10.1103/PhysRevLett.100.181301)

PACS numbers: 98.70.Vc, 98.80.Es

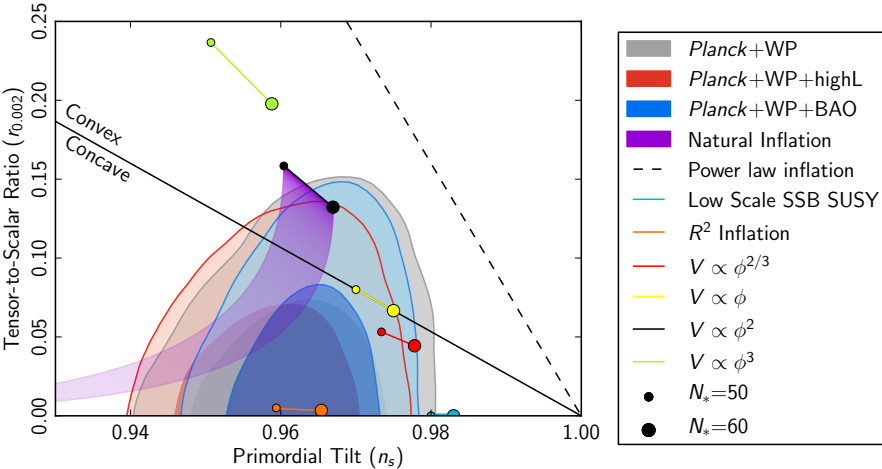
## Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity

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The Planck nominal mission cosmic microwave background (CMB) maps yield unprecedented constraints on primordial non-Gaussianity (NG). Using three optimal bispectrum estimators, separable template-fitting (KSW), binned, and modal, we obtain consistent values for the primordial local, equilateral, and orthogonal bispectrum amplitudes, quoting as our final result  $f_{NL}^{local} = 2.7 \pm 5.8$ ,  $f_{NL}^{equil} = -42 \pm 75$ , and  $f_{NL}^{ortho} = -25 \pm 39$  (68% CL statistical); and we find the integrated Sachs-Wolfe lensing bispectrum expected in the  $\Lambda$ CDM scenario. The results are based on comprehensive cross-validation of these estimators on Gaussian and non-Gaussian simulations, are stable across component separation techniques, pass an extensive suite of tests, and are confirmed by skew- $C_l$ , wavelet bispectrum and Minkowski functional estimators. Beyond estimates of individual shape amplitudes, we present model-independent, three-dimensional reconstructions of the Planck CMB bispectrum and thus derive constraints on early-Universe scenarios that generate primordial NG, including general single-field models of inflation, excited initial states (non-Bunch-Davies vacua), and directionally-dependent vector models. We provide an initial survey of scale-dependent feature and resonance models. These results bound both general single-field and multi-field model parameter ranges, such as the speed of sound,  $c_s \geq 0.02(95\%CL)$ , in an effective field theory parametrization, and the curvaton decay fraction  $r_D \geq 0.15(95\%CL)$ . **The Planck data put severe pressure on ekpyrotic/cyclic scenarios.** The amplitude of the four-point function in the local model  $\tau_{NL} < 2800(95\%CL)$ . Taken together, these constraints represent the highest precision tests to date of physical mechanisms for the origin of cosmic structure.



# Implications for inflation—summary plot



# Constraints on isocurvature modes

34

Planck Collaboration: Constraints on inflation

| Model  | $\beta_{\text{iso}}(k_{\text{low}})$ | $\beta_{\text{iso}}(k_{\text{mid}})$ | $\beta_{\text{iso}}(k_{\text{high}})$ | $\alpha_{\text{RR}}^{(2,2500)}$ | $\alpha_{\text{II}}^{(2,2500)}$ | $\alpha_{\text{RI}}^{(2,2500)}$ | $\Delta n$ | $-2\Delta \ln \mathcal{L}_{\text{max}}$ |
|--|--------------------------------------|--------------------------------------|---------------------------------------|---------------------------------|---------------------------------|---------------------------------|------------|---|
| General model:   |                                      |                                      |                                       |                                 |                                 |                                 |            |   |
| CDM isocurvature   | 0.075                                | 0.39                                 | 0.60                                  | [0.98:1.07]                     | 0.039                           | [-0.093:0.014]                  | 4          | -4.6                                    |
| ND isocurvature  | 0.27                                 | 0.27                                 | 0.32                                  | [0.99:1.09]                     | 0.093                           | [-0.18:0]                       | 4          | -4.2                                    |
| NV isocurvature  | 0.18                                 | 0.14                                 | 0.17                                  | [0.96:1.05]                     | 0.068                           | [-0.090:0.026]                  | 4          | -2.5                                    |
| Special CDM isocurvature cases:                                  |                                      |                                      |                                       |                                 |                                 |                                 |            |   |
| Uncorrelated, $n_{\text{II}} = 1$ , ("axion")                    | 0.036                                | 0.039                                | 0.040                                 | [0.98:1]                        | 0.016                           | –                               | 1          | 0                                       |
| Fully correlated, $n_{\text{II}} = n_{\text{RR}}$ , ("curvaton") | 0.0025                               | 0.0025                               | 0.0025                                | [0.97:1]                        | 0.0011                          | [0:0.028]                       | 1          | 0                                       |
| Fully anti-correlated, $n_{\text{II}} = -n_{\text{RR}}$          | 0.0087                               | 0.0087                               | 0.0087                                | [1:1.06]                        | 0.0046                          | [-0.067:0]                      | 1          | -1.3                                    |

# Statistical Isotropy?

**Table 23.** Summary of dipole modulation likelihood results as a function of scale for all four *Planck* CMB solutions.

| Data set  | FWHM [°] | A                         | ( <i>l</i> , <i>b</i> ) [°] | $\Delta \ln \mathcal{L}$ | Significance |
|-----------|----------|---------------------------|-----------------------------|--------------------------|--------------|
| Commander | 5        | $0.078^{+0.020}_{-0.021}$ | (227, -15) ± 19             | 8.8                      | 3.5 $\sigma$ |
| NILC      | 5        | $0.069^{+0.020}_{-0.021}$ | (226, -16) ± 22             | 7.1                      | 3.0 $\sigma$ |
| SEVEM     | 5        | $0.066^{+0.021}_{-0.021}$ | (227, -16) ± 24             | 6.7                      | 2.9 $\sigma$ |
| SMICA     | 5        | $0.065^{+0.021}_{-0.021}$ | (226, -17) ± 24             | 6.6                      | 2.9 $\sigma$ |
| WMAP5 ILC | 4.5      | $0.072 \pm 0.022$         | (224, -22) ± 24             | 7.3                      | 3.3 $\sigma$ |
| Commander | 6        | $0.076^{+0.024}_{-0.025}$ | (223, -16) ± 25             | 6.4                      | 2.8 $\sigma$ |
| NILC      | 6        | $0.062^{+0.025}_{-0.026}$ | (223, -19) ± 38             | 4.7                      | 2.3 $\sigma$ |
| SEVEM     | 6        | $0.060^{+0.025}_{-0.026}$ | (225, -19) ± 40             | 4.6                      | 2.2 $\sigma$ |
| SMICA     | 6        | $0.058^{+0.025}_{-0.027}$ | (223, -21) ± 43             | 4.2                      | 2.1 $\sigma$ |
| Commander | 7        | $0.062^{+0.028}_{-0.030}$ | (223, -8) ± 45              | 4.0                      | 2.0 $\sigma$ |
| NILC      | 7        | $0.055^{+0.029}_{-0.030}$ | (225, -10) ± 53             | 3.4                      | 1.7 $\sigma$ |
| SEVEM     | 7        | $0.055^{+0.029}_{-0.030}$ | (226, -10) ± 54             | 3.3                      | 1.7 $\sigma$ |
| SMICA     | 7        | $0.048^{+0.029}_{-0.029}$ | (226, -11) ± 58             | 2.8                      | 1.5 $\sigma$ |
| Commander | 8        | $0.043^{+0.032}_{-0.029}$ | (218, -15) ± 62             | 2.1                      | 1.2 $\sigma$ |
| NILC      | 8        | $0.049^{+0.032}_{-0.031}$ | (223, -16) ± 59             | 2.5                      | 1.4 $\sigma$ |
| SEVEM     | 8        | $0.050^{+0.032}_{-0.031}$ | (223, -15) ± 60             | 2.5                      | 1.4 $\sigma$ |
| SMICA     | 8        | $0.041^{+0.032}_{-0.029}$ | (225, -16) ± 63             | 2.0                      | 1.1 $\sigma$ |
| Commander | 9        | $0.068^{+0.035}_{-0.037}$ | (210, -24) ± 52             | 3.3                      | 1.7 $\sigma$ |
| NILC      | 9        | $0.076^{+0.035}_{-0.037}$ | (216, -25) ± 45             | 3.9                      | 1.9 $\sigma$ |
| SEVEM     | 9        | $0.078^{+0.035}_{-0.037}$ | (215, -24) ± 43             | 4.0                      | 2.0 $\sigma$ |
| SMICA     | 9        | $0.070^{+0.035}_{-0.037}$ | (216, -25) ± 50             | 3.4                      | 1.8 $\sigma$ |
| WMAP3 ILC | 9        | 0.114                     | (225, -27)                  | 6.1                      | 2.8 $\sigma$ |
| Commander | 10       | $0.092^{+0.037}_{-0.040}$ | (215, -29) ± 38             | 4.5                      | 2.2 $\sigma$ |
| NILC      | 10       | $0.098^{+0.037}_{-0.039}$ | (217, -29) ± 33             | 5.0                      | 2.3 $\sigma$ |
| SEVEM     | 10       | $0.103^{+0.037}_{-0.039}$ | (217, -28) ± 30             | 5.4                      | 2.5 $\sigma$ |
| SMICA     | 10       | $0.094^{+0.037}_{-0.040}$ | (218, -29) ± 37             | 4.6                      | 2.2 $\sigma$ |