# High Energy Collisions of BHs



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Phys.Rev.Lett.96:071301,2006 (arXiv: hep-th/0512002)
Phys.Rev.Lett.101:161101,2008 (arXiv:0806.1738)
Phys.Rev.Lett., 2009 (arXiv:0907.1252)


New Worlds in Astroparticle Physics, São Tomé (8-10 September)

# Very High Energy Collisions


**Gravity ends up winning since energy gravitates**

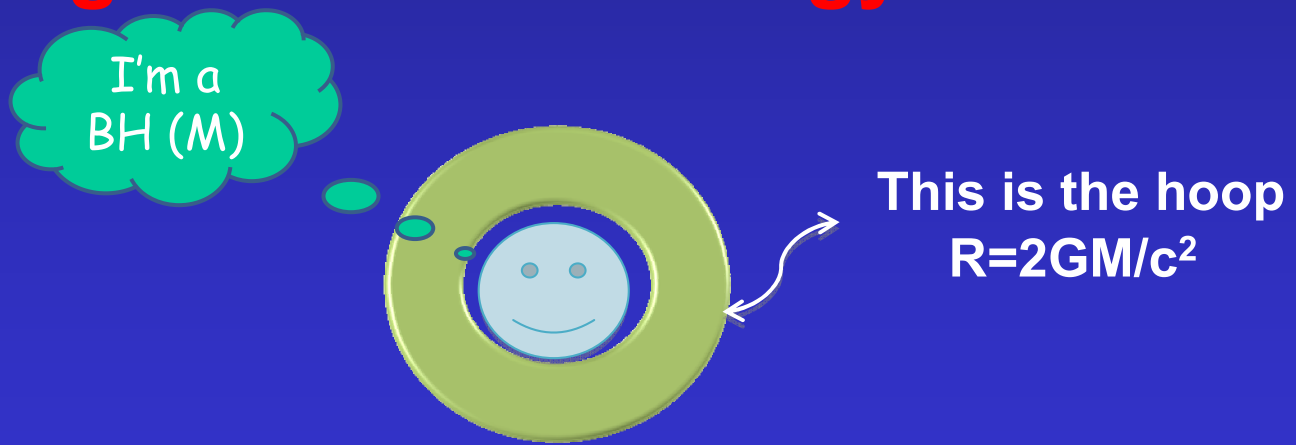
**The end of particle physics as we know it?**

# Hoop Conjecture

**(Thorne 1972)**

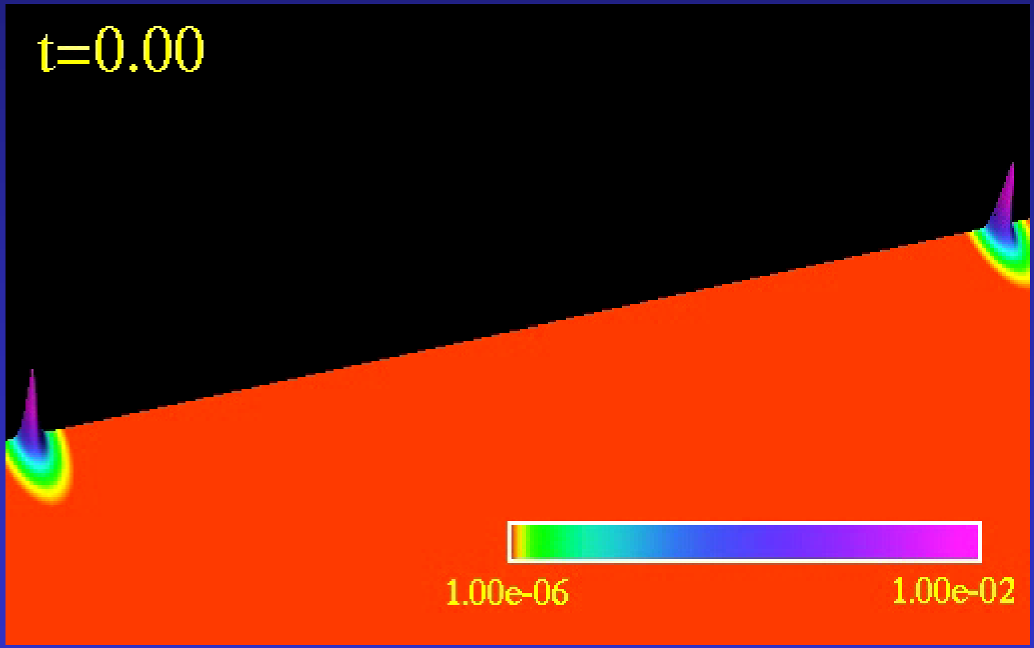
- "An imploding object forms a BH when, and only when, a circular hoop with a certain critical circumference can be made that encloses the object in all directions. The critical circumference is $2\pi$ the Schwarzschild radius of the object."

## Large amount of energy in small region



# Hoop Conjecture

**(Thorne 1972)**




Choptuik & Pretorius, arXiv:0908.1780

# Conclusions…

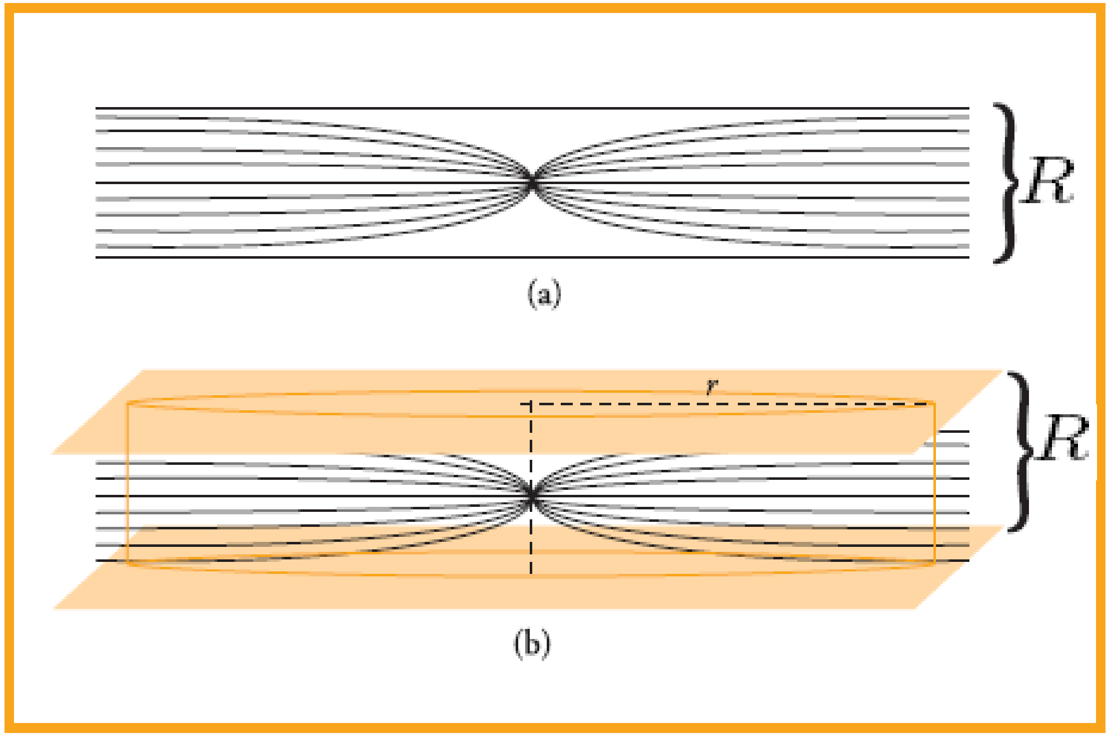
Black holes do form generically in high-energy collisions

Transplanckian scattering is likely to be well described by BH scattering!

The ultra-relativistic 2-body scattering problem is a fascinating theoretical problem within GR, though becomes an issue of more pressing concern if the true Planck scale is within reach of near future experiments...

# Brane-New World

- Braneworlds open new possibilities




$$r^{n+2} \times \mathcal{G} = G_{3+n} M$$

$$\mathcal{G} = \frac{G_{3+n} M}{r^{n+2}}$$

$$4\pi r^2 \times R^n \times \mathcal{G} = G_{3+n} M$$

$$\mathcal{G} = \frac{G_{3+n} M}{4\pi R^n r^2}$$

$$\mathcal{G} = \frac{G_3 M}{r^2}$$

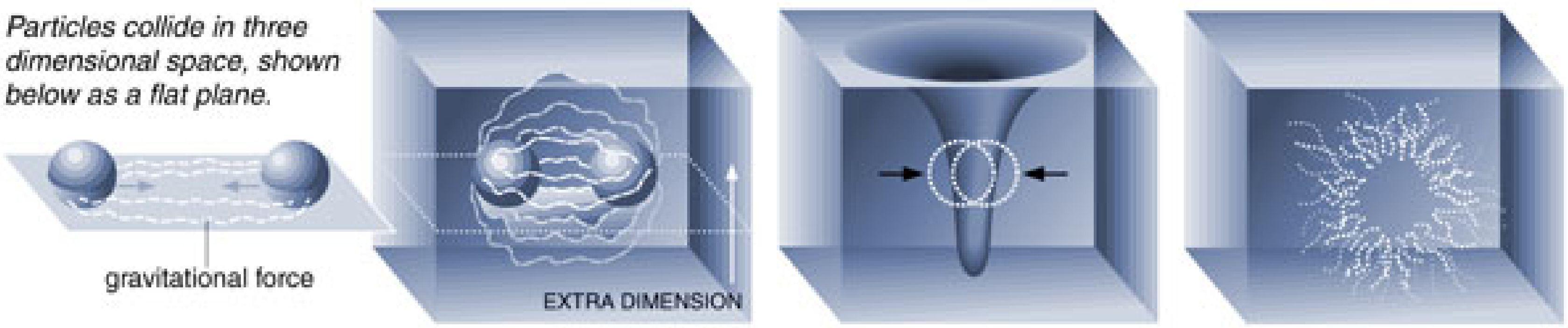
At large distances, one recovers Newton's gravity!

# High-Energy Collisions of BHs

- If Planck scale is small, may be useful for accelerators and Cosmic Ray Experiments!
- Compute production cross section, radiated energies, etc

## Black Holes on Demand

Scientists are exploring the possibility of producing miniature black holes on demand by smashing particles together. Their plans hinge on the theory that the universe contains more than the three dimensions of everyday life. Here's the idea:




As the particles approach in a particle accelerator, their gravitational attraction increases steadily.

When the particles are extremely close, they may enter space with more dimensions, shown above as a cube.

The extra dimensions would allow gravity to increase more rapidly so a black hole can form.

Such a black hole would immediately evaporate, sending out a unique pattern of radiation.

NYT, 9/11/01

# High-Energy Collisions of BHs

- Cosmic Censorship: do horizons always form?

- Universal Dyson limit on maximum luminosity
  (F. Dyson '63)

- Is there critical behavior at the onset of BH formation?
  (F. Pretorius '07)

- Test analytical techniques, their predictions and power
  (Penrose '74, D'Eath & Payne '93, Eardley & Giddings '02, Cardoso & Lemos '02)

- Holography and HIC
  (Amsel et al '08; Gubser et al '08)

# What was known so far (for head-on collisions)

# I. BH formation: analytics for head-on

- Superpose two Aichelburg-Sexl metrics

  ✓ Future trapped surface on shock-wave collision
  (Penrose '74, Eardley & Giddings '02)

  ✓ Max. gravitational radiation: 29% $\gamma\, M_0$

**Changes ST from Petrov type D to N**

**No handle on bounds**

**No details on wave emission**

- Perturb superposed A-S metric, correction: 16% $\gamma\, M_0$
  (D'Eath & Payne '90s)

# II. Radiation: ZFL

(Weinberg '64; Smarr '77)

Idea: Infinite acceleration, zero duration process

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + h_{\mu\nu}\,, \\ h_{\mu\nu} &= 4\int \frac{S_{\mu\nu}(t_r, x')}{\|x' - x\|} d^3x'\,, \\ S_{\mu\nu} &= T_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} T \end{aligned}$$

Take two free particles, changing abruptly at t=0

$$\begin{aligned} T^{\mu\nu} = \sum_{i=1,2} & \frac{P_i^\mu P_i^\nu}{E_i}\delta^3(x - v_i t)\theta(-t) \\ & + \frac{P_i^{'\mu} P_i^{'\nu}}{E_i'}\delta^3(x - v_i' t)\theta(t) \end{aligned}$$

# II. Radiation: ZFL

(Weinberg '64; Smarr '77)

$$\frac{dE}{d\omega d\Omega} = \frac{M^2\gamma^2 v^4}{\pi^2}\frac{\sin^4\theta}{(1-v^2\cos^2\theta)^2}$$

i) Flat, large memory ( $[h_{t=\infty}-h_{t=-\infty}]^2 \propto dE/d\omega$)

ii) Radiation isotropic in the limit $\gamma \rightarrow \infty$

iii) Functional relation $E_{rad}(\gamma)$

v) Roughly 65% of maximum possible at $\gamma=3$

With cutoff $M\gamma\omega \approx 0.4$ we get 25% efficiency for conversion of gws

**Flat background, linearized approximation!**

# III. Radiation: PP

(Cardoso & Lemos '02)

Metric=Schwarzschild + small perturbation due to infalling particle

$$h_{\mu\nu} = \begin{bmatrix} H_0(r)f(r) & H_1(r) & 0 & 0 \\ H_1(r) & H_2(r)/f(r) & 0 & 0 \\ 0 & 0 & r^2K(r) & 0 \\ 0 & 0 & 0 & r^2K(r)\sin^2\theta \end{bmatrix} e^{-i\omega t}Y_{l0}(\theta)$$

$$\frac{d^2\Psi}{dr_*^2} + \left[\omega^2 - V\right]\Psi = \left(1 - \frac{2M}{r}\right)S(\omega, r)$$

$$\frac{V}{1-2M/r} = \frac{2\lambda^2(\lambda+1)r^3 + 6\lambda^2r^2M + 18\lambda rM^2 + 18M^3}{r^3(3M+\lambda r)^2},$$

$$S(\omega, r) = \frac{4i\mu\lambda\sqrt{4l+2}}{\omega(3M+\lambda r)^2}e^{-i\omega r_*}, \quad \lambda \equiv \frac{(l-1)(l+2)}{2},$$

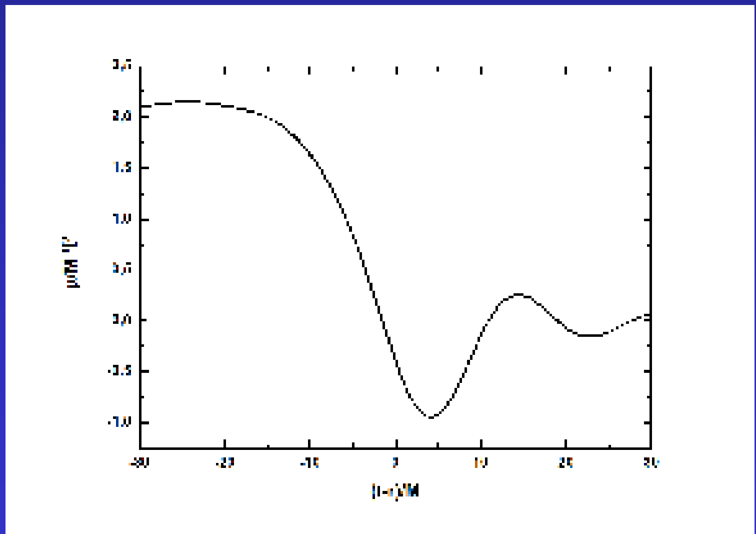
# III. Radiation: PP

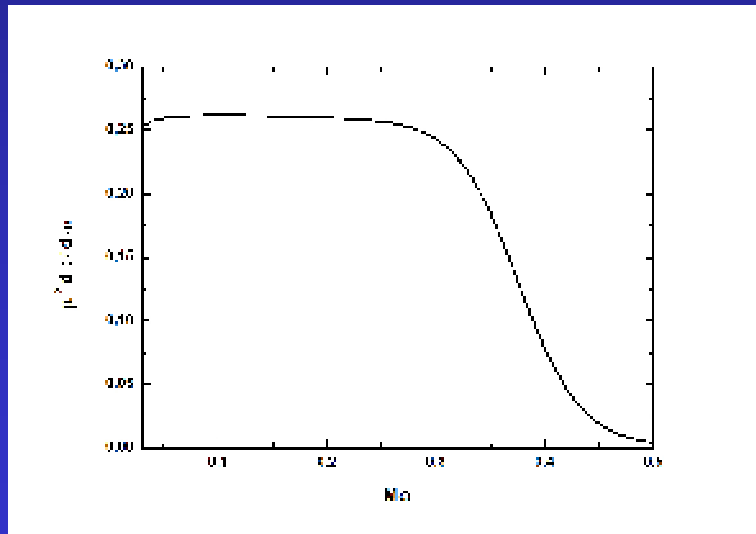
(Cardoso & Lemos '02)

i) dE/d$\omega$ flat at sufficiently low $\omega$, large memory

ii) Functional relation $E_{rad}(\gamma)$

iii) Roughly 65% of maximum possible at $\gamma$=3





**Linearized, PP approximation!**

# A very brief history

- Pioneers: Hahn and Lindquist 1960s, Eppley and Smarr 1970s

- Numerical simulations Grand Challenge ’90s

  Limited accuracy, mild boosts

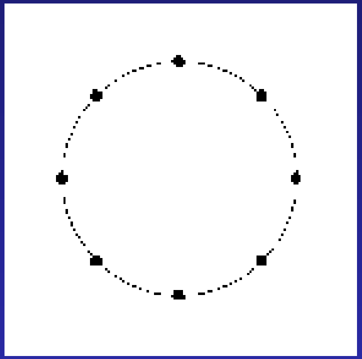
- Breakthrough: Pretorius 2005, Brownsville, NASA Goddard 2005

# Numerical simulations

- LEAN code Sperhake '07
  - ✓ Based on the Cactus computational toolkit
  - ✓ BSSN formulation (ADM-like, but strongly hyperbolic)
  - ✓ Puncture initial data Brandt & Brügmann 1996
  - ✓ Elliptic solver: TwoPunctures Ansorg 2005
  - ✓ Mesh refinement: Carpet Schnetter '04
- Numerically very challenging!
  - ✓ Length scales: $M_{ADM} \propto \gamma M_0$
  - ✓ Horizon Lorentz-contracted "Pancake"
  - ✓ Mergers extremely violent
  - ✓ Substantial amounts of unphysical "junk" radiation

# Gravitational Waves

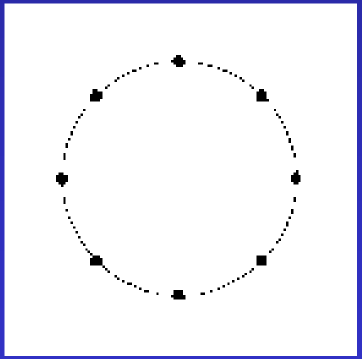
- Most important result: Emitted gravitational waves (GWs)
- Newman-Penrose scalar



$\Psi_4 = C_{\alpha\beta\gamma\delta}\, n^\alpha\, m^\beta\, n^\gamma\, m^\delta$

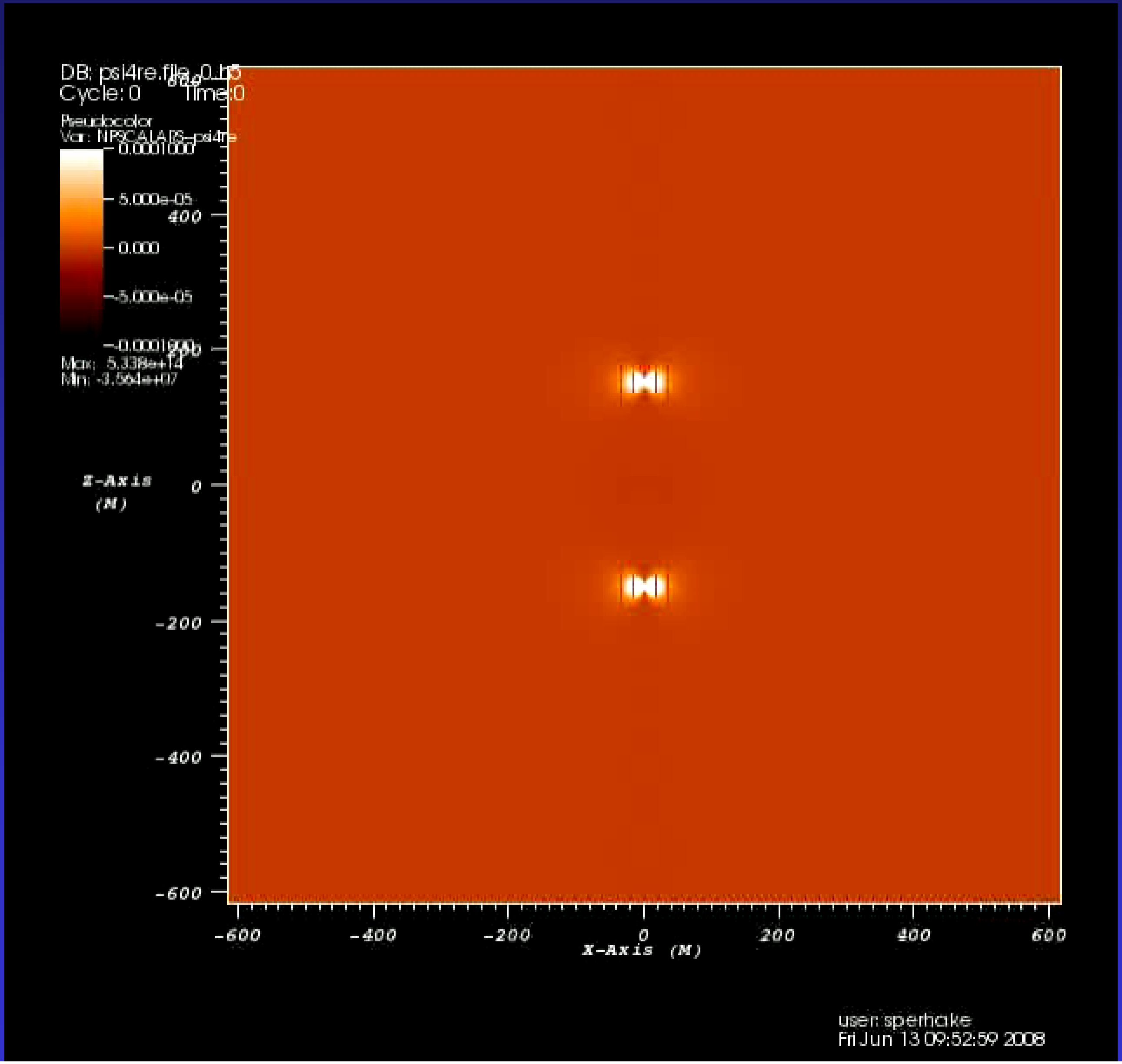
Complex $\longrightarrow$ 2 free functions

- GWs allow us to measure



  - ✓ Radiated energy $E_{rad}$
  - ✓ Radiated momenta $P_{rad}$, $J_{rad}$
  - ✓ Angular dependence of radiation
  - ✓ Predicted strain $h_+$, $h_x$

$\beta=0.93$



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Cycle: 0 Time:0
Pseudocolor
Var: NPSCALARS-psi4re
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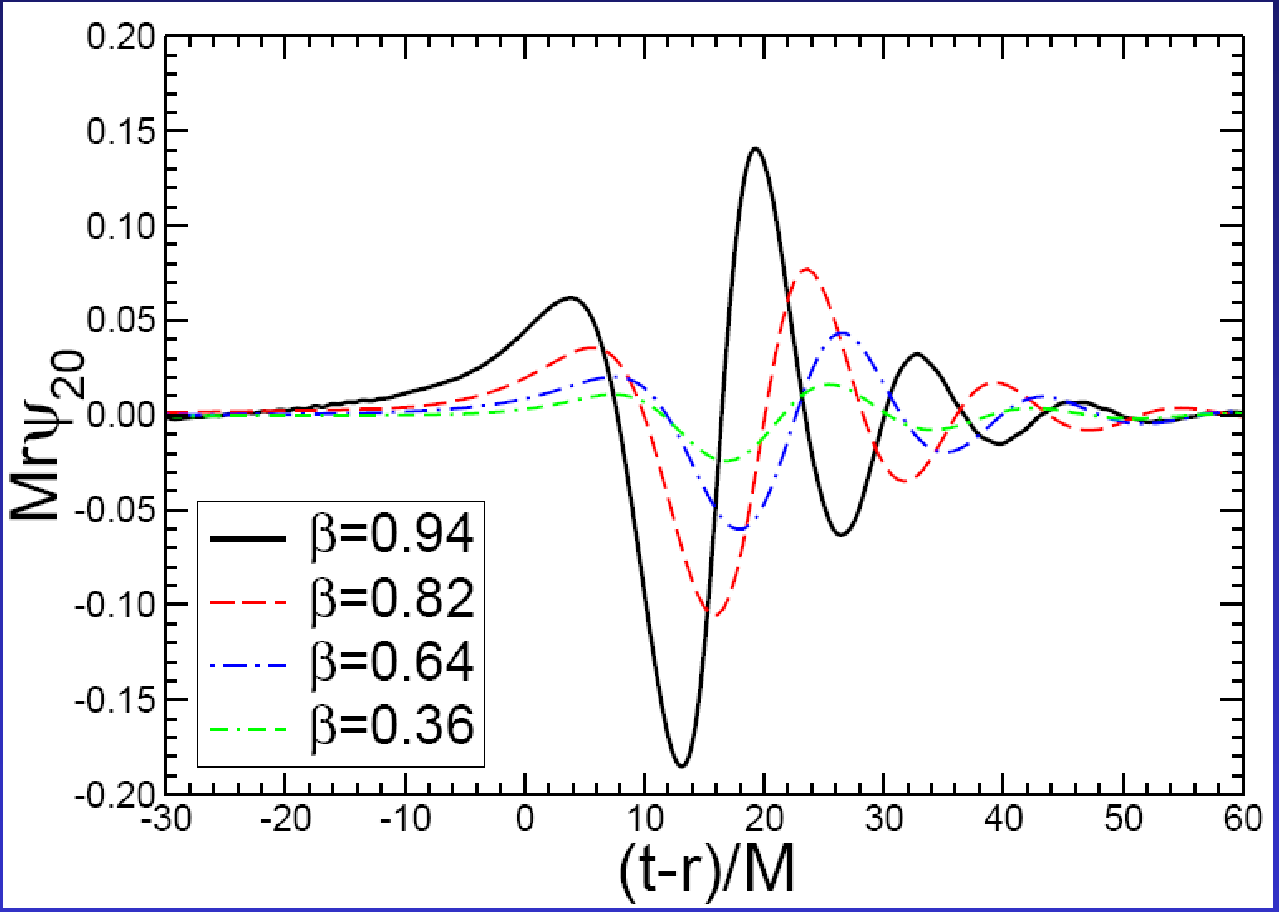
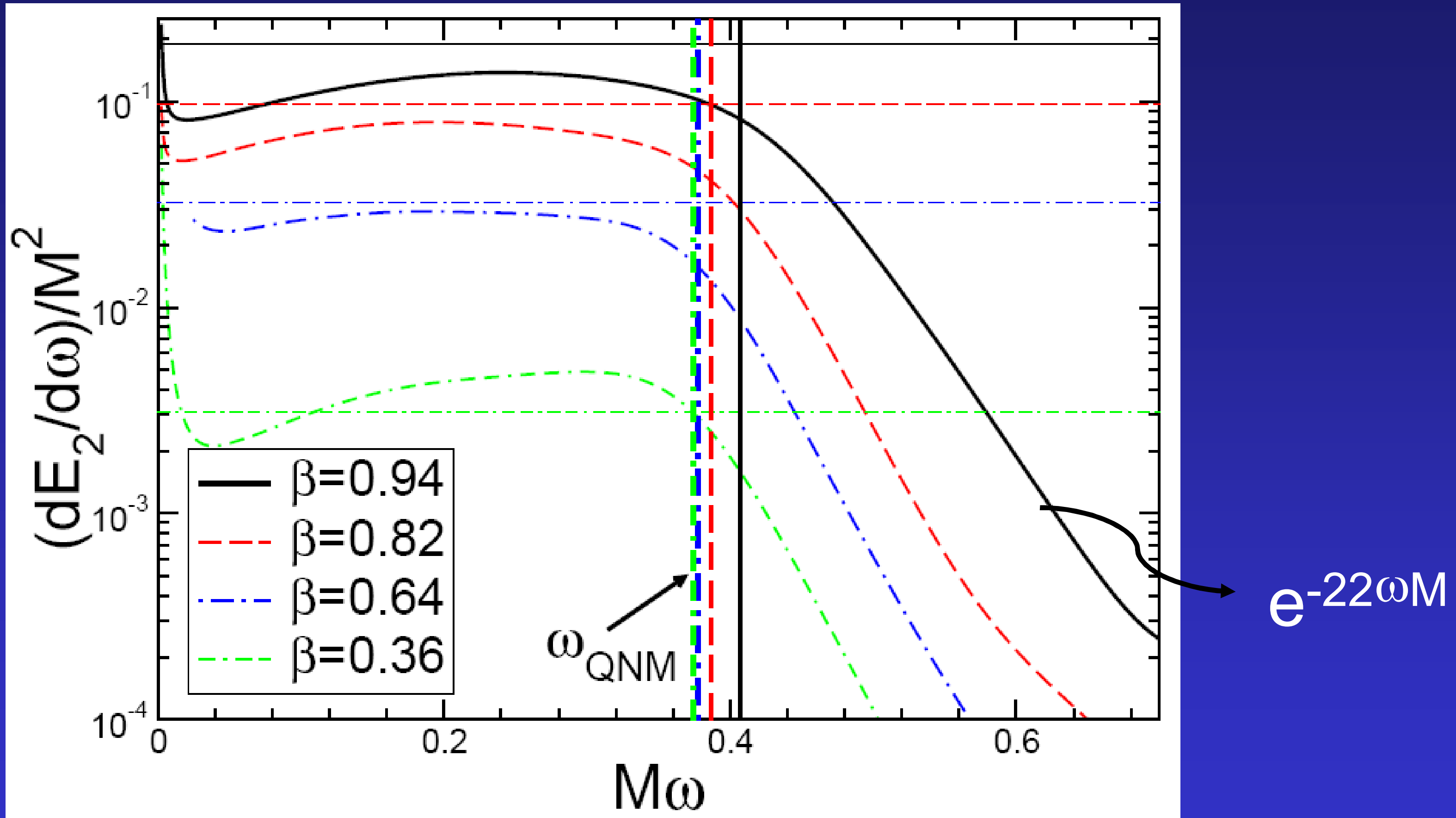
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 5.338e+14
Min: -3.564e+07
600
400
200
0
-200
-400
-600
Z-Axis (M)
-600
-400
-200
0
200
400
600
X-Axis (M)
user: sperhake
Fri Jun 13 09:52:59 2008

# Results



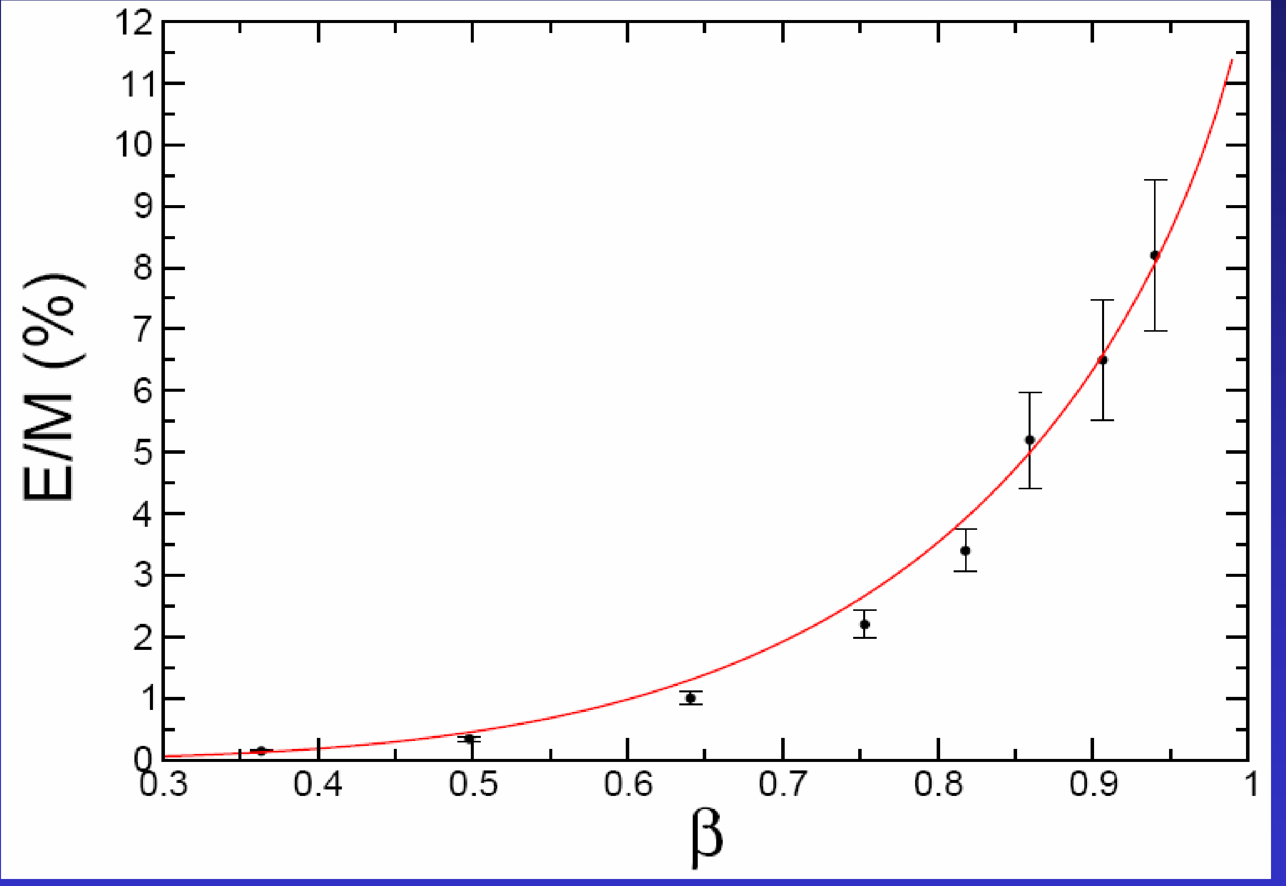
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
Mrψ20
-30
-20
-10
0
10
20
30
40
50
60
(t-r)/M
β=0.94
β=0.82
β=0.64
β=0.36

# Results



$(dE_2/d\omega)/M^2$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
0
0.2
0.4
0.6
$M\omega$
β=0.94
β=0.82
β=0.64
β=0.36
$\omega_{QNM}$
$e^{-22\omega M}$

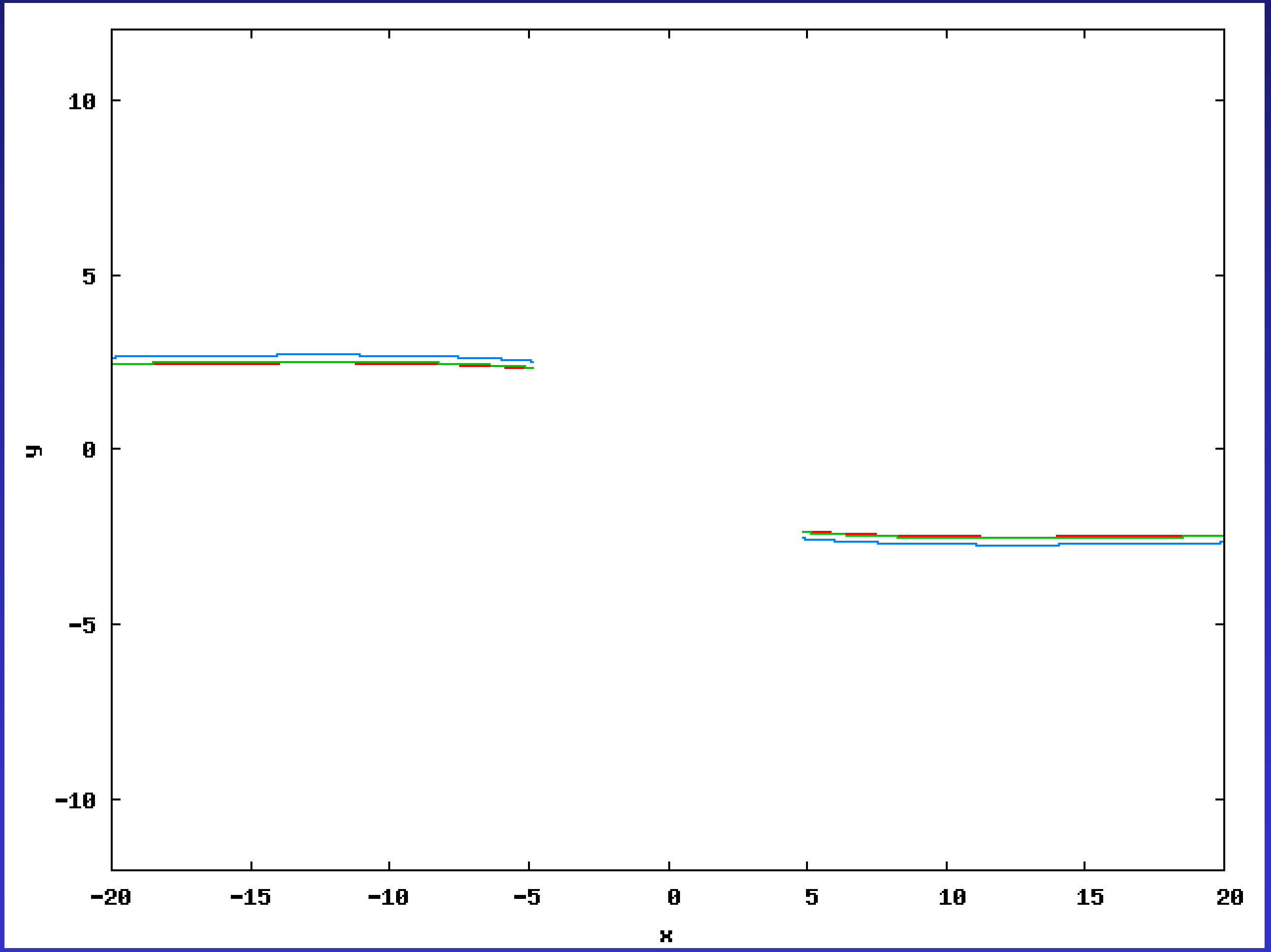
# Results



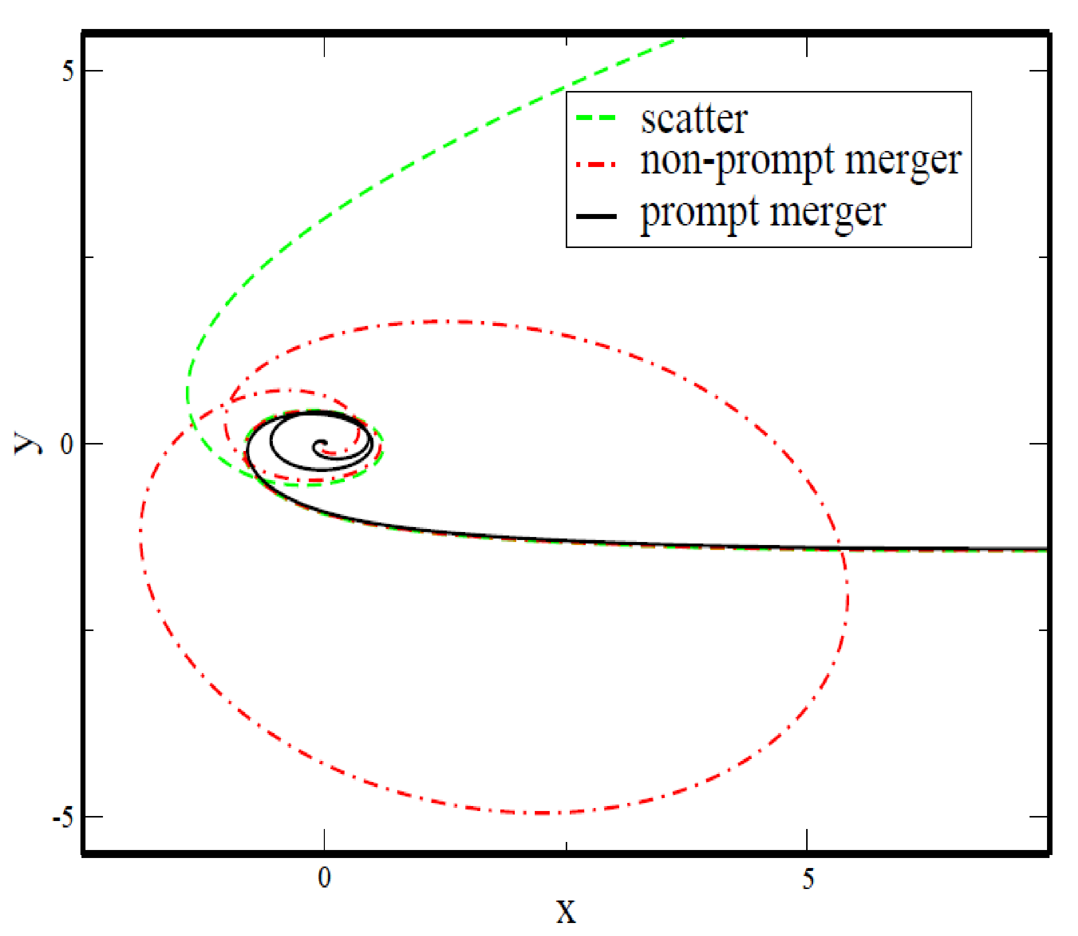

14%

$$\frac{E}{M} = E_\infty \left( \frac{1+2\gamma^2}{2\gamma^2} + \frac{(1-4\gamma^2)\log\left(\gamma+\sqrt{\gamma^2-1}\right)}{2\gamma^3\sqrt{\gamma^2-1}} \right)$$

# Grazing collisions



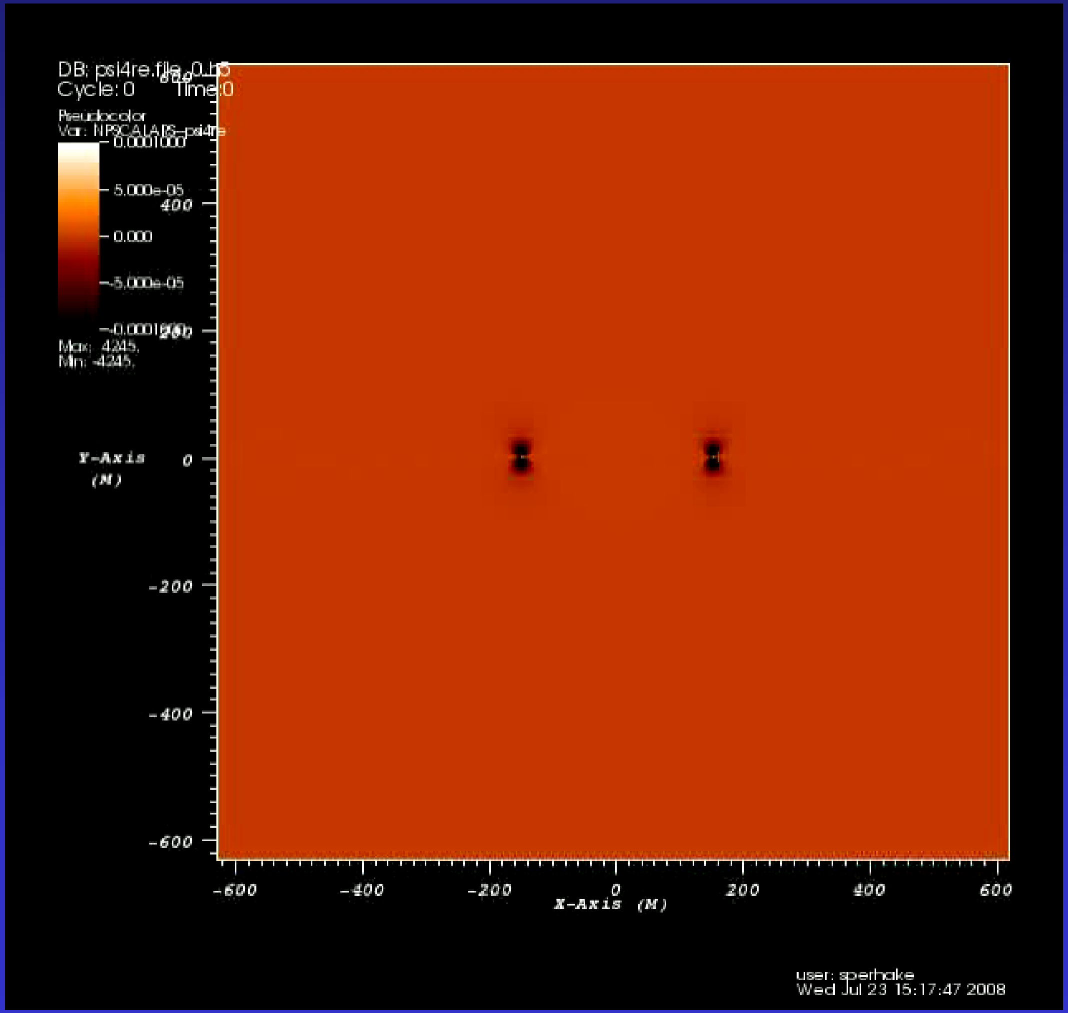
# Grazing collisions



# Grazing collisions

✓ Non head-on: production cross-section

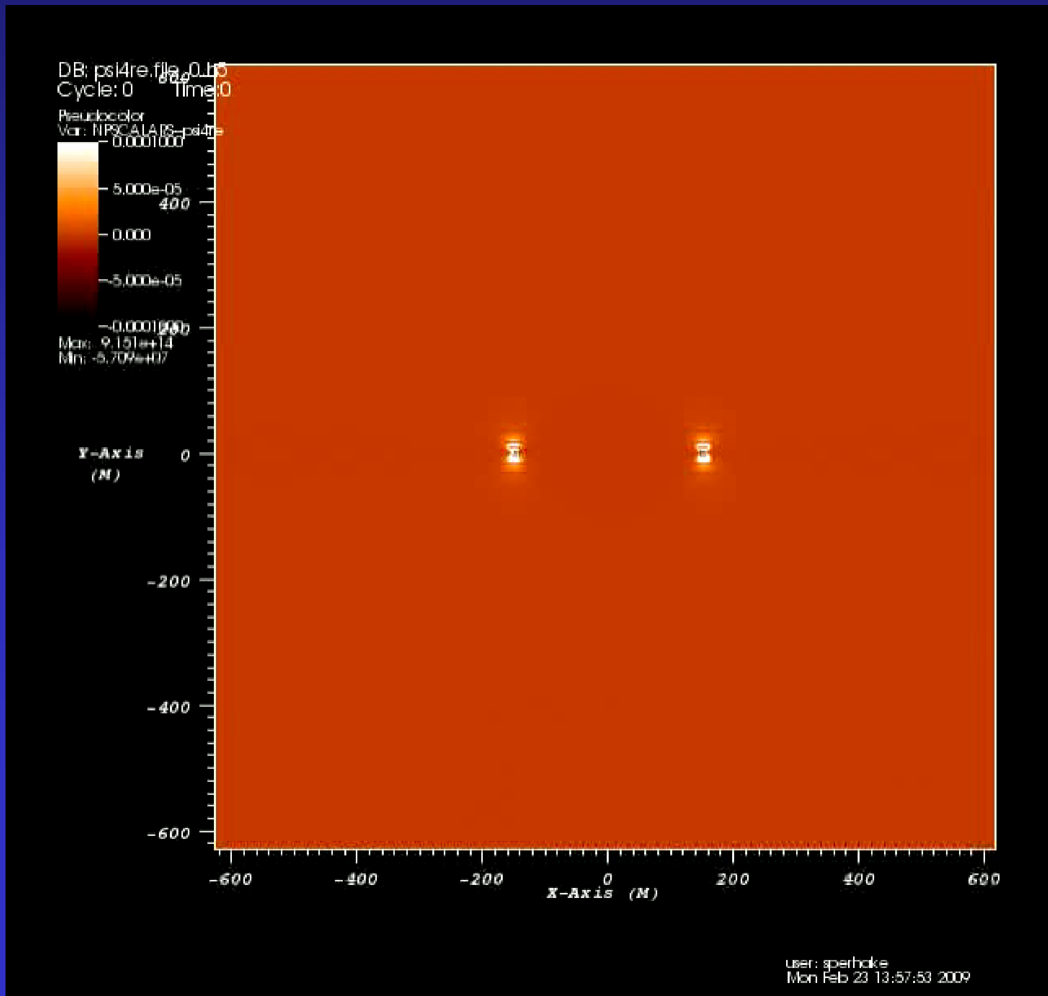
$\beta$=0.75



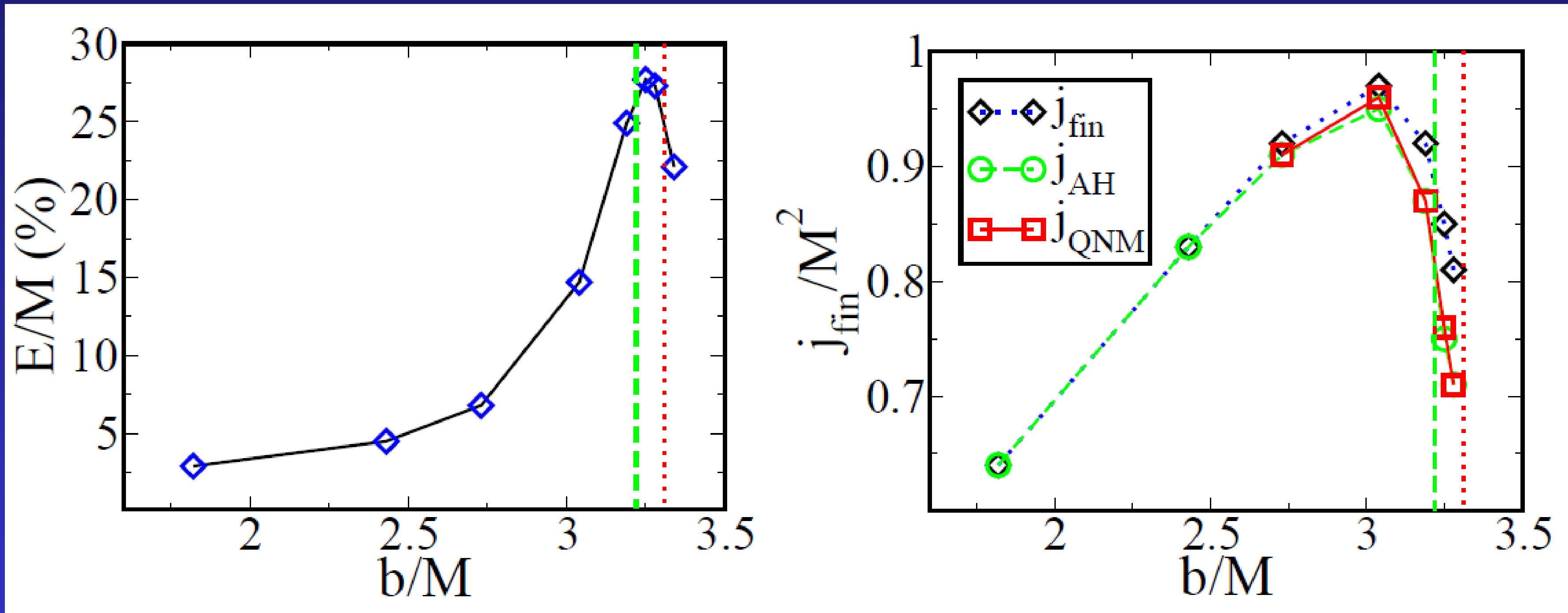
# Grazing collisions

✓ Non head-on: production cross-section

$\beta$=0.75



# Grazing collisions




More than 25%CM energy radiated for v=0.75 c!

Final BH rapidly spinning

# Grazing collisions

- ✓ Non head-on: production cross-section

$$b/M=2.5/v$$

Huge luminosities (almost Dyson limit)

# Future work

- ✓ Higher dimensions, other backgrounds
- ✓ Spin, charge
- ✓ Particles with structure (Choptuik & Pretorius)
- ✓ Can the end product be radiation alone?
- ✓ Can the end product be more than one hole?
- ✓ Quantum effects

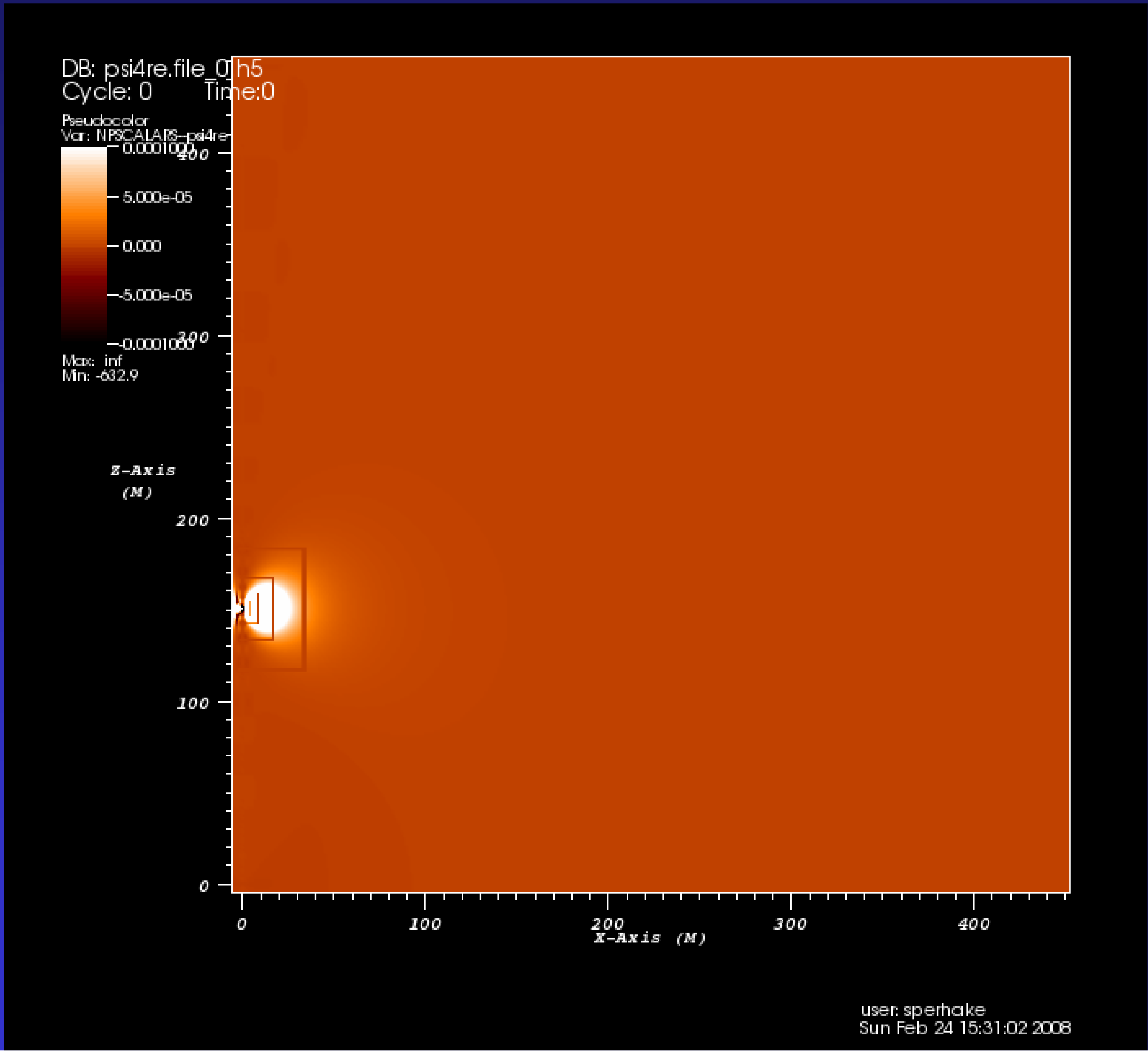
*Thank you*

# A list of tasks

- Target: Predict time evolution of BBH in GR
- Einstein equations:
  - ✓ Cast as evolution system
  - ✓ Choose specific formulation
  - ✓ Discretize for Computer
- Choose coordinate conditions: Gauge
- Fix technical aspects:
  - ✓ Mesh-refinement / spectral domains
  - ✓ Excision
  - ✓ Parallelization
  - ✓ Find large computer
- Construct realistic initial data
- Start evolution and wait…extract physics from data

Gourgoulhon gr-qc/0703035

# $T = 0\, M_0$



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Pseudocolor
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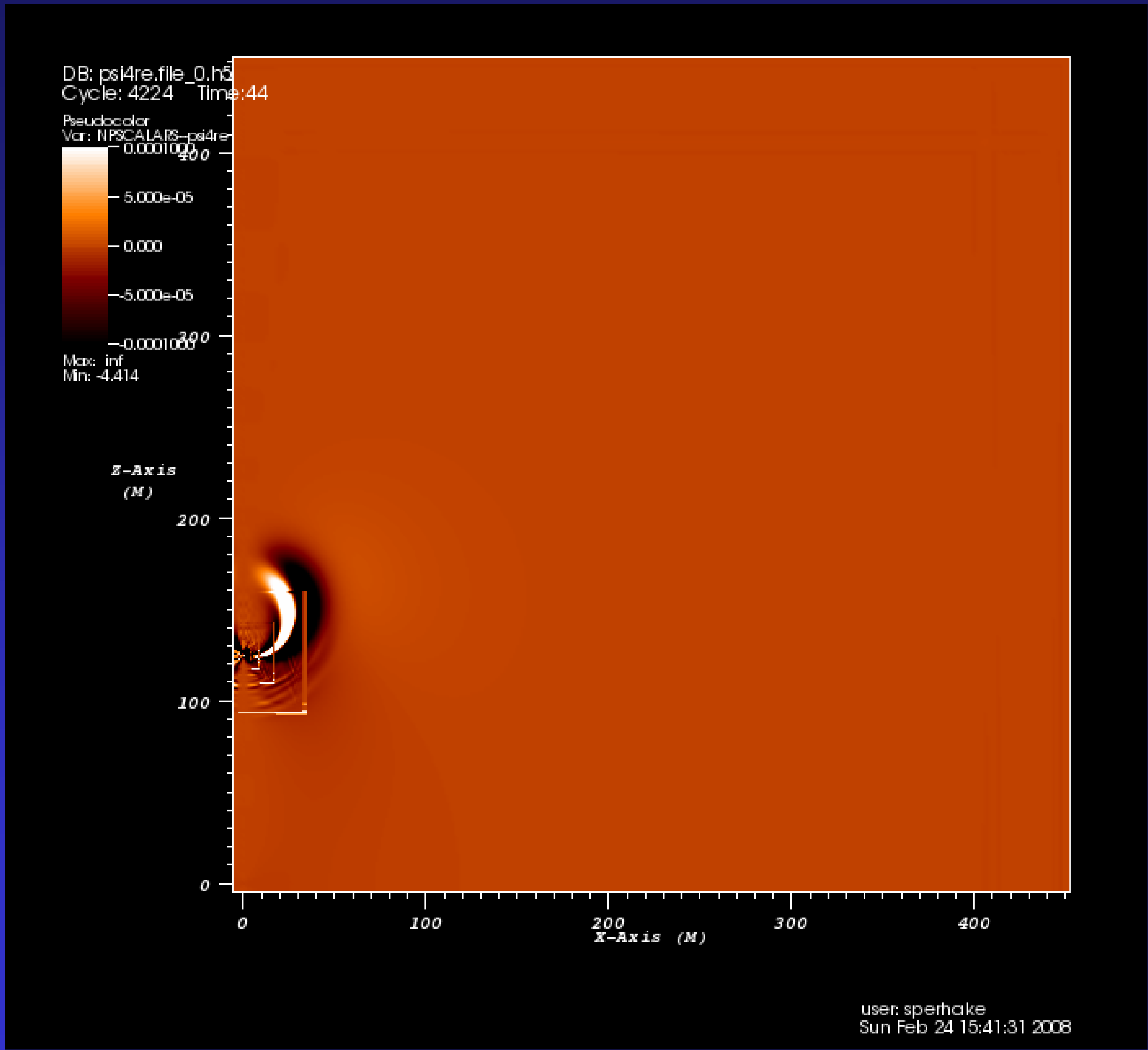
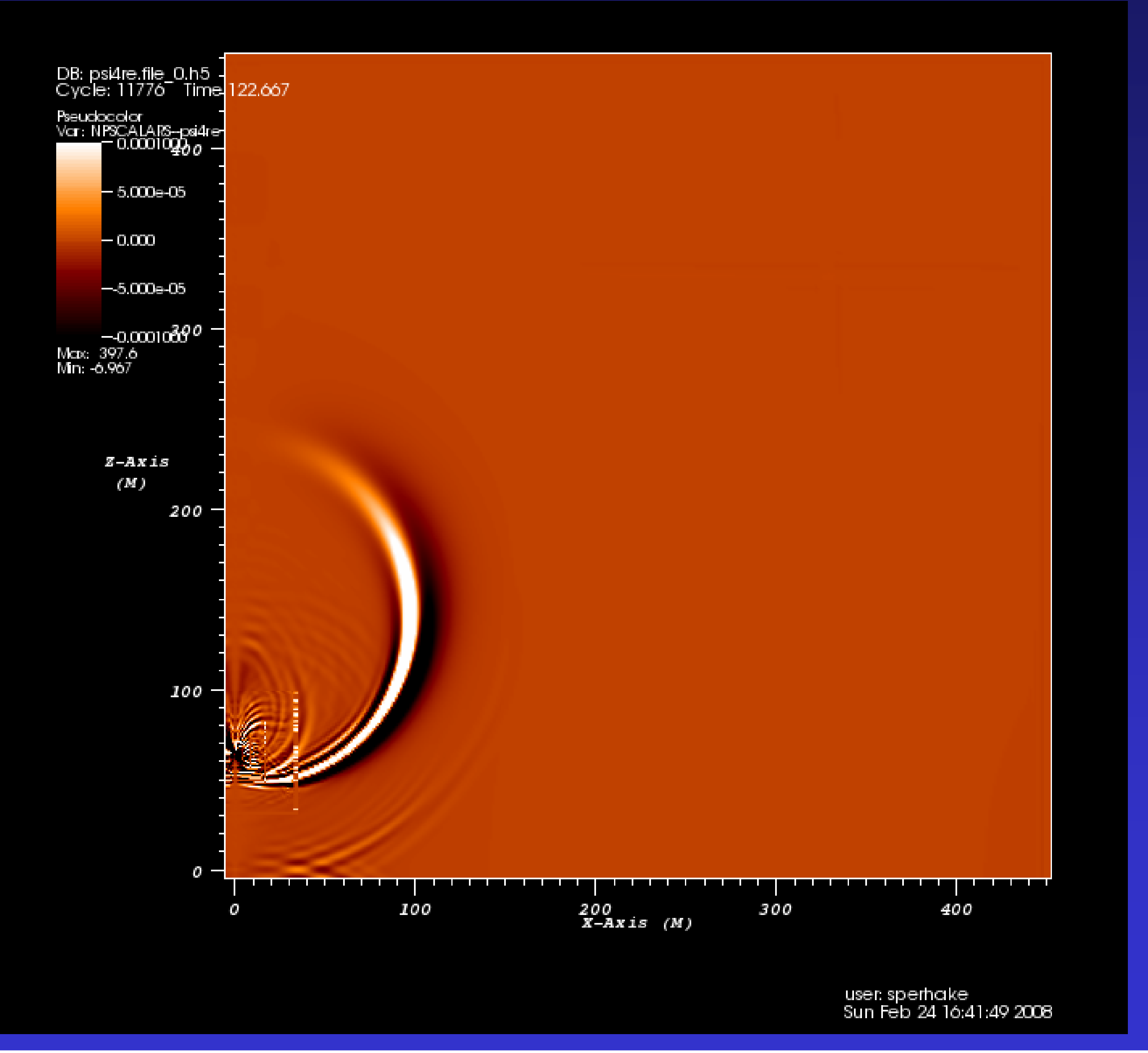
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: inf
Min: -632.9
Z-Axis
(M)
400
300
200
100
0
0
100
200
300
400
X-Axis (M)
user: sperhake
Sun Feb 24 15:31:02 2008

$$T = 29.6\, M_0$$


DB: psi4re.file_0.h5
Cycle: 4224 Time:44
Pseudocolor
Var: NPSCALARS-psi4re
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: inf
Min: -4.414
Z-Axis (M)
X-Axis (M)
0
100
200
300
400
user: sperhake
Sun Feb 24 15:41:31 2008

# $T = 82.6\, M_0$



DB: psi4re.file_0.h5
Cycle: 11776 Time: 122.667
Pseudocolor
Var: NPSCALARS-psi4re
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 397.6
Min: -6.967
Z-Axis (M)
X-Axis (M)
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100
200
300
400
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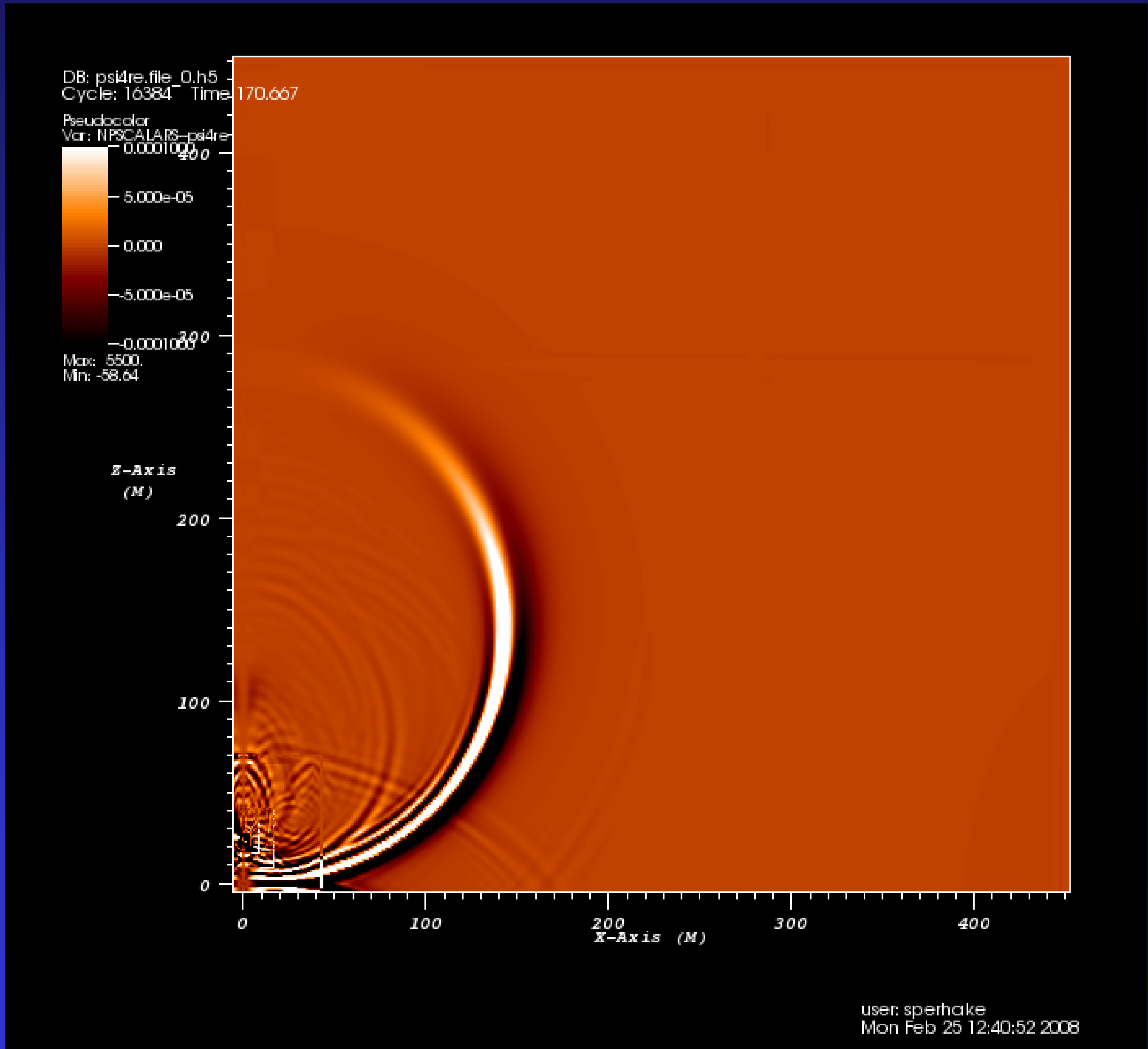
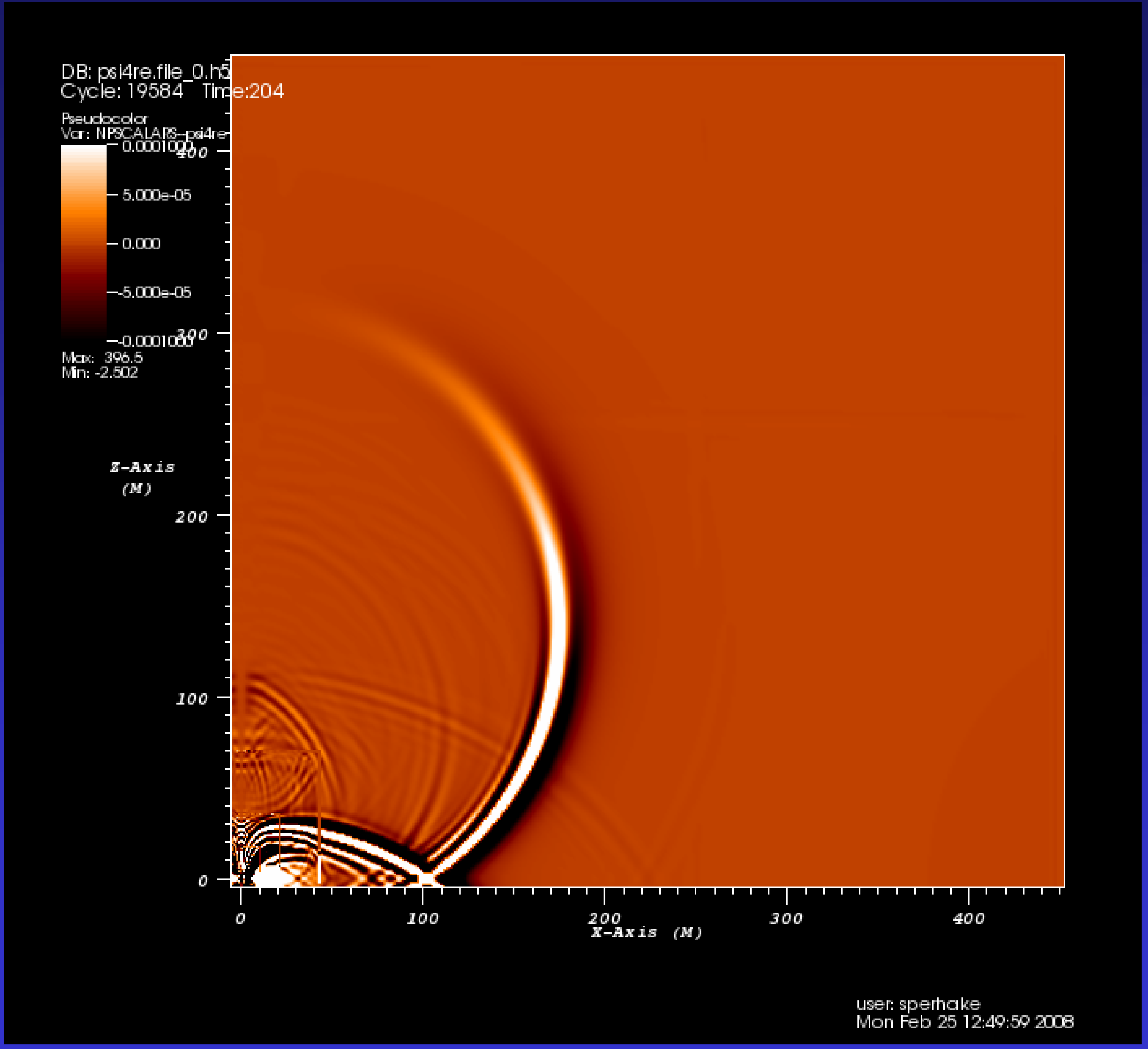
# $T = 114.9\, M_0$

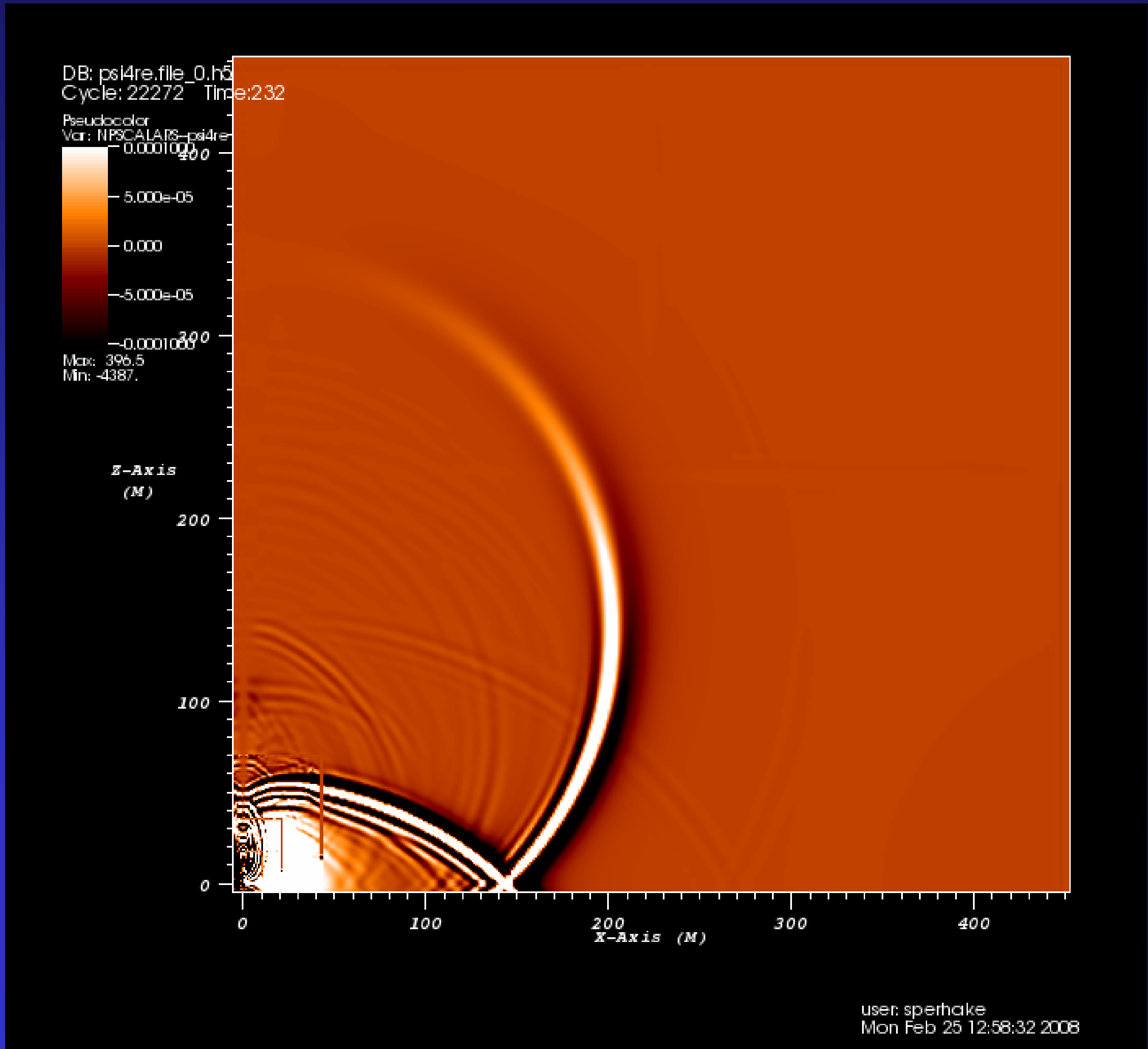

DB: psi4re.file_0.h5
Cycle: 16384 Time:170.667
Pseudocolor
Var: NPSCALARS-psi4re
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 5500.
Min: -58.64
Z-Axis (M)
X-Axis (M)
0
100
200
300
400
user: sperhake
Mon Feb 25 12:40:52 2008

# $T = 137.3\, M_0$



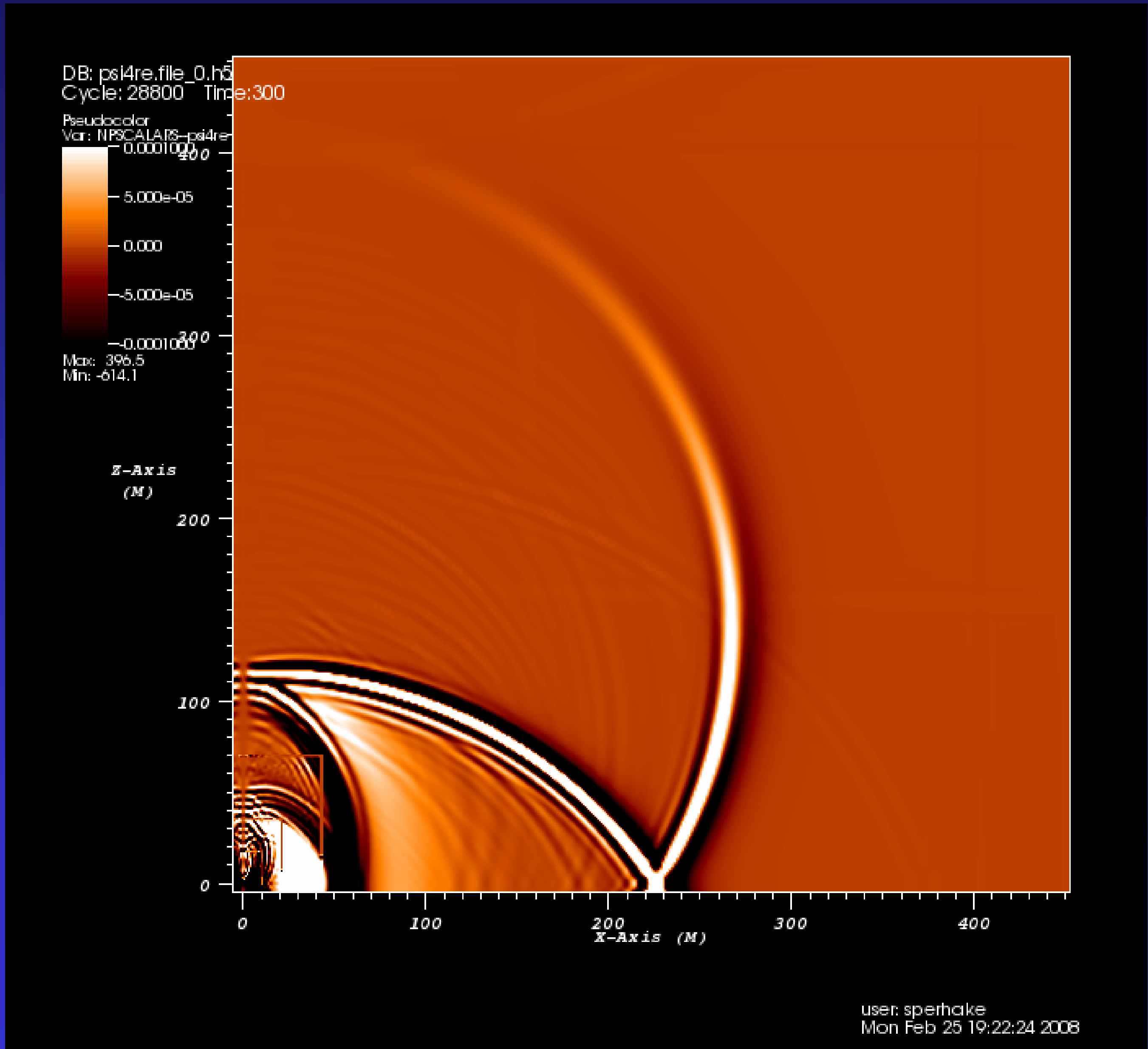
DB: psi4re.file_0.h5
Cycle: 19584 Time:204
Pseudocolor
Var: NPSCALARS–psi4re
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 396.5
Min: -2.502
Z-Axis
(M)
400
300
200
100
0
0
100
200
300
400
X-Axis (M)
user: sperhake
Mon Feb 25 12:49:59 2008

$$T = 156.1 M_0$$



DB: psi4re.file_0.h5
Cycle: 22272 Time:232
Pseudocolor
Var: NPSCALARS-psi4re
0.0001000
5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 396.5
Min: -4387.
Z-Axis (M)
400
300
200
100
0
X-Axis (M)
0
100
200
300
400
user: sperhake
Mon Feb 25 12:58:32 2008

# $T = 201.9\, M_0$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Cycle: 28800 Time:300
Pseudocolor
Var: NPSCALARS-psi4re
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5.000e-05
0.000
-5.000e-05
-0.0001000
Max: 396.5
Min: -614.1
Z-Axis (M)
X-Axis (M)
0
100
200
300
400
user: sperhake
Mon Feb 25 19:22:24 2008

# Grazing collisions